ANALYSIS OF WAVE RESTORING FORCES DUE TO OSCILLATION OF A RECTANGULAR BARGE AND THEIR EFFECTS ON CONSTRUCTION OF OFFSHORE STRUCTURES

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DECLARATION AND RECOMMENDATIONS

Declaration

This thesis is my original work and has not been presented in this University or any other for the award of a degree.

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Recommendation

This thesis has been submitted for examination with our approval as the University supervisors.

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DEDICATION

I dedicate this thesis to my loving father Mr. Kipsang Malel, my brother Kenneth Kiplangat Sang, my siblings and the entire family. Your consistent prayers and words of inspiration have been a great contribution to my academic achievements thus far.

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ABSTRACT

Waves are a common phenomenon in the ocean environment and are caused when energy is transmitted across the water surface and their effects have prompted studies on hydrodynamic loads on offshore structures. When interactions of waves occur with offshore structures, there is damping of wave loads, which are added mass, damping coefficients and restoring forces and moments, on them. Most researchers have worked on restoring forces and moments of oscillating bodies with forward speed in 2 dimensions. However, little has been done with regard to an oscillating rectangular barge with zero forward speed in a 3-dimensional axis. This study therefore analyzed the wave restoring forces due to oscillation of a rectangular barge and their effects on the construction of offshore structures. To do this, it was necessary that the solution of the velocity potential of the outgoing waves be derived from the radiation potential after which the added mass and the damping coefficients were derived. This aided in the derivation and analysis of the restoring forces of the oscillating rectangular barge. From the velocity potential, the series form of the Greens function was used since its results asymptotically converged more easily than those of the integral form and at the same time eliminated irregular frequencies. Additionally, the wave characters of the outgoing waves were predicted from those of the incoming waves. The simulation process was done using the Fortran software to generate data which was later run in the origin software to generate graphs for further discussion. The findings in this thesis will aid in reducing the computational effort required in the design of offshore floating structures, especially at phases where load instances must be considered, and ensures modelling of offshore structures with high restorative forces. This research can further be extended to analysis of wave restoring moments for a rectangular barge oscillating at zero forward speed.

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LIST OF ABBREVIATIONS AND ACRONYMS

BEM	Boundary Element Method.
BBC	Bottom Boundary Condition.
DFSBC	Dynamic Free Surface Boundary Condition.
KFSBC	Kinematic Free Surface Boundary Condition.
FEM	Finite Element Method

LIST OF SYMBOLS

B	Position vector in the (x, y, z)
C	Wave speed
F	Restoring force
g	Gravitational acceleration
h	Water depth
k	Wave number
P	Atmospheric pressure
n	Unit normal vector to the body surface
t	Time
Q	Wave amplitude
Re	Velocity potential's real part
S_B	Body surface
ω	Wave's angular frequency
ρ	Water density
λ	Wavelength of the wave
η	Free surface wave elevation
(x, y, z)	Cartesian plane
$\phi(x, y, z)$	Velocity potential
ϕ_I	Incident velocity potential
ϕ_j	Solution of the radiation potential
ξ_j	Representation of body motion amplitude
J_0	Bessel zero order function

CHAPTER ONE
INTRODUCTION

1.1 Background Information

Waves occur normally in oceans and are caused when energy is transmitted across the water surface. Ocean engineers have long been careful to take into account the factors of slamming and deck wetness, among other characteristics in the construction process, to ensure that a vessel can resist the tough sea conditions. Offshore structures are crucial in the production of gas and electricity as well as to the drilling and production of oil (Shukla & Karki, 2016). Among other important applications of these structures are the loading and offloading of cargo from ships (Abyn *et al.,* 2014).

According to Moan (1997), it is crucial to make sure that structures are modelled in a way that will lessen or even eliminate structural failures while also accurately presenting high production levels over extended periods of time. To meet their needs, it is important to make sure that these structures are extremely productive while also operating in a way that doesn't endanger the lives of other marine structures (Rouhan & Schoefs, 2003). In this sense, greater emphasis should be placed during the building process on features that will guarantee that these vessels can endure challenging circumstances for extended periods of time.

Research has established that waves cause loads on most offshore fixed and floating platforms. Tromans and Vanderschuren (2017) defines wave loading to be a random process which is caused on offshore structures regardless of their position; that is, whether located on the shores or off the shores. Interactions of offshore structures with surface waves normally results in the creation of periodic loads around these structures. Faltinsen (1990) states that understanding wave-induced loads is critical and should be incorporated into the design and operation of offshore structures. According to Ngina *et al.* (2015), added mass, damping and restoring forces and moments are among the periodic loads whose causal agents are interactions of waves with structures. While damping and added mass have been the subject of several research, the examination of restoring forces and moments has received less attention. An essential feature that should be taken into account while designing offshore buildings is the analysis of restoring forces.

Bhattacharyya (1978) analyzed the non-linear roll motion of a ship with significant use of polynomials of restoring forces. Regardless of using structures of different shapes in his study, the study majorly focused on the restoring forces with regard to the roll motion. Cardo *et al.* (1984) also focused on the effects of non-linear restoring forces for a coupled motion where the focus was on a structure oscillating in three degrees of freedom. This study will otherwise put more emphasis on the analysis of restoring forces on the surge motion.

Talyan (1999) approximated the restoring forces of a non-linear roll model using a generalized asymptotic method. The main approach that was put in place to govern this study was the time domain approach. Surendran *et al*. (2003) paid attention to general performance of a ship in the sea and sought the solutions of restoring forces and moments using the MATLAB program where the effects of roll motion were only analyzed on the roll motion.

Pesman and Taylan (2012) examined the stability characteristics on roll motion in the main parametric resonance regions. Here, the variations on restoring forces and moments were modelled with respect to time and to the instantaneous roll angle. The roll response was then evaluated using the frequency domain approach. Ahmed *et al.* (2016) used an integral scheme to detect both the excitation forces and the restoring forces of a forced non- linear oscillation simultaneously. Kianejad *et al.* (2018) analyzed the restoring moments of a ship in dynamic condition which was then measured with basis on heave motion and on the respective location of the wave's crest and trough. They pointed out that the calculation of restoring moments increases the overall accuracy in the process of simulating a ship's stability.

Yu *et al.* (2019) conducted a quantitative prediction of the restoring forces in the roll motion of a container within its 5 degrees of freedom. However, this study majorly concentrated on a rectangular shaped barge and analysis of its restoring forces. A number of researchers have been taking specifications in relation to the properties of the medium that is being studied. In this instance, it was expected that the medium's density, water, would remain constant. It is similarly significant to highlight that, as in other research investigations of the same kind, a single-layer fluid technique was used in this study. Other than that, the structures used differed; in this instance, a rectangular structure was used. Furthermore, in contrast to prior research that have been employing the time domain method, the frequency domain approach was used.

This study, therefore, focused on the analysis of restoring forces where the wave characters were analyzed and used in predicting the wave restoring forces. That is, structures operating off the shores should be modelled with incorporation of high restoring forces. Which is due to the fact that an increase in distance away from the shores causes a subsequent increase in wave amplitudes, which increases frequencies. Increased frequencies and amplitudes demand that the structures should have high restoring forces to ensure that accidents are not experienced. The study assumed that there exists a similarity between the wave characters of the incoming and the outgoing waves and thus the incoming wave characters were used as a prediction of the outgoing wave characters. Initially, Ngina *et al.* (2015) used wave characters to analyze wave exciting forces of an oscillating rectangular barge. In addition, Juma *et al.* (2020) analyzed the added mass and damping coefficients of an oscillating rectangular box with use of radiation velocity potential. This work therefore, added value to the works of these scholars by portraying the relationship between wave characters and added mass and damping coefficients of outgoing waves in the analysis of restoring forces of an oscillating barge at zero forward speed.

1.2 Statement of the Problem

Occurrence of waves in the ocean environment are common and are known to be of different types with different causal mechanisms. Waves have wide impacts on offshore structures which operates in the ocean including capsizing, destruction, sweeping away and crucial cases of deaths caused by tsunami-like waves. Furthermore, different forces and moments are created around offshore platforms as a result of the interactions between them and the waves. Numerous periodic loads have been extensively studied, including the analysis of damping coefficients and added mass, but the analysis of restoring forces and moments has received less attention. Understanding restoring forces and moments is crucial for modelling offshore structures so that they maintain their equilibrium position even when subjected to powerful waves. It is known that there have been cases of offshore structures getting washed away by waves and many researchers have worked on these problems. Nevertheless, their work has not been sufficient as it is evident that more of these cases keep occurring. Furthermore, there are a lot of wave effects on these structures which need to be looked into independently and in better ways. Therefore, the wave restoring forces for a rectangular body moving with a zero-forward speed and stationery at a finite depth were examined in this study. In contrast to earlier research that employed a two-dimensional technique, this work fully embraced a three-dimensional approach for restoring forces prediction. The prediction of wave restoring forces was the main goal here, however incident wave characteristics were used to predict outgoing wave characteristics, and coefficients of restoring forces were derived using damping coefficients and added mass.

1.3 Objectives

1.3.1 General Objective

To model the wave restoring forces from the outgoing waves due to a freely floating body on a uniformly distributed fluid and analyze their input in design of offshore structures.

1.3.2 Specific Objectives

i) To investigate the incident waves in order to predict the outgoing waves due to restoring acceleration.

ii) To analyze added mass and damping coefficients of the outgoing waves so as to obtain restoring forces.
iii) To predict restoring forces of the outgoing waves in order to relate with construction of offshore structures.

1.4 Assumptions

i) The fluid under investigation is considered to be incompressible and homogenous with a constant density.
ii) The fluid under study is considered to be irrotational and the flow field is conservative.
iii) The sea bottom is assumed to be impermeable and stationary.

1.5 Justification

For oceanographers and ocean engineers, the study of wave-induced motions is crucial. There have been advancements in technology and the researchers have come up with new structures whose performance are quite better than the previous ones. With this, knowledge of wave induced loads are highly crucial. In this area of study, a lot has been done around added mass and damping. Otherwise, works on restoring forces has remained unworked for over time. Incorporating the aspects of restoring forces in modelling of structures is important as it ensures that the modelled structures will, at all time, work amidst the chaotic nature of its environment and within its stable position. Additionally, more research has been done on cylindrical shapes, although hydrodynamic load analysis on rectangular shapes is just as significant. This is caused by the fact that offshore structures come in a variety of designs, some of which are rectangular and have not been thoroughly researched. Even when developing rectangular structures for mineral collection in the oceans, understanding restorative forces is crucial. It is much simpler to extract more oil and other minerals when these structures are stable and in their equilibrium positions. The findings of this research will have significant implications for oceanographers and ocean engineers. This is because they will understand how restorative forces affect a structure. This will also improve and boost mining and extraction of minerals and other valuables from the sea and increase revenue which will help this great country, Kenya, achieve its 2030 vision.

1.6 Definition of Terms

Wave – In a given medium, a wave is a disturbance that transfers energy from one place to another.

Incident wave- A wave that is travelling from a source towards a structure and which eventually makes it to oscillate.

Capsizing- This is the effect of a structure overturning in water.

Outgoing wave- This is a wave generated from a source due to effects of oscillation. These waves travel away from the structure.

Frequency- This is the rate in which a wave passes through a given point over a specific period of time.

Exciting force- This is the force that is responsible for the motion of once stationery body.

Added mass- This is the inertia that is added to a system when it deflects some volume of the fluid surrounding it when subjected to some force.

Damping – This is the influence of an oscillatory system which reduces, restricts or prevents the oscillation of the structure.

Restoring force- This is a force that is responsible for bringing an oscillating structure back to its equilibrium position.

Wavelength- This is the distance between one wave crest to the other in a wave.

Wave celerity- This refers to the speed in which an individual wave propagates.

Offshore structures- Are structures built to operate deep in the sea and are used for activities such as oil and gas exploration.

Oscillations- This describes how a structure moves back and forth between two deformation points.

CHAPTER TWO
LITERATURE REVIEW

2.1 Ocean Waves

In oceans, with water as the medium, the wind transfers energy from one point of the ocean to the other as it blows. Simultaneously, little ripples are created, which gradually enlarge and become waves (Cruz, 2007). According to Toffoli and Bitner-Gregersen (2017), a wave requires an initial equilibrium condition, which is broken by certain early disturbances and replaced by a restoring force. There are multiple types of waves, often occurring simultaneously, and they vary in their progression. These are the waves produced by wind, moving ships and gravitational disturbances from the sun and moon (Massel, 1996). Conversely, restorative forces are often driven by gravity and surface tension (Toffoli & Bitner-Gregersen, 2017).

When waves are formed, they travel in a variety of directions, and their energies are consumed due to a variety of factors, including bottom friction, angular dispersion, air or water turbulence, and spreading across greater areas (Thorpe, 2005). This is the reason why these waves get shorter as they move away from where they originated. Waves become sharp and unstable when they are opposed because their height increases and their length decreases. Greater steepness is associated with higher water particle velocities. The water particles escape and wave breaking occurs when such speeds surpass the wave's velocity (Deo, 2013).

Major cases of capsizing and tilting of offshore structures have been reported in the recent past. Among them is the accident on the X-Press Pearl ship which occurred in 2021 in Sri Lanka. De Vos *et al.* (2021) reported that the ship was loaded with cargo and on board were tones of nitric acid, oil and plastic. Due to the harsh conditions from the sea waves, the ship lost its stability and started swaying from side to side. This then made the corrosive and highly flammable nitric acid to leak and accidentally catch fire making the ship to catch fire and sink. Had the knowledge on restoring forces been incorporated fully in the modelling of X-Press Pearl ship, there would have been minimal chances of the accident occurring. A similar case is the tilting of an offshore wind platform in China which is reported to have tilted due to the action of strong waves on it. According to Asim *et al.* (2022), the structure was tilted by the waves and thus making it slide and sink. The accident could have been avoided had the knowledge of restoring forces been incorporated in its design.

According to Mendes *et al.* (2021), the analysis of periodic loads and their effects on offshore structures is very crucial in modelling of dependable and resilient offshore structures. They pointed out in their study that in the analysis of stress and loads on offshore wind turbines, considerations on dynamic forces caused by waves should be incorporated. These forces, including restoring forces and moments, dictates the general workability of these structures and ensures the safety of lives on-board.

While damping coefficients and added mass have been studied, little research has been done on restoring forces and moments. According to Newton's third law of motion, for every action, there is an equal and opposite reaction. Therefore, for every excitation force, there is an equal and opposite restoring force. This is the force that acts on an oscillating body acted upon by a wave with an aim of bringing it back to its equilibrium position. In addition, force is a product of mass and acceleration and this brings the relationship between added mass, wave characters and the restoring forces. This study therefore analyzed wave restoring forces by borrowing the idea of wave characters and linking its relation to added mass and damping.

For high performances of offshore structures, Tang *et al.* (2021) pointed out that it is crucial to put into consideration the effects of restoring forces in modelling of ships moving at a forward speed. Their study pointed out that the study of restoring forces gives improvements in predicting of the ship's accuracy especially under complicated conditions such as those with higher forward speed. Nonetheless, most researchers in the same field have found the problem of analyzing these forces for a body in forward speed as being engaging and results not agreeing with earlier works from other researchers. Therefore, it is important to analyze the same forces by putting into place the aspect of zero forward speed.

With advancements in technology, there have been modelling of structures of different shapes. The ocean engineers have recently come up with offshore structures of varied geometry and this includes the rectangular shaped structures. With these advancements, researchers in this field have been keen to analyze restoring forces and moments, as a hydrodynamic load, acting on offshore structures. Tanaka and Hudspeth (1988) are among the researchers who had interest in analyzing the restoring forces and moments of offshore structures. However, their main structures of interest were circular cylinders which were forced to oscillate under the action of earthquakes. Awal *et al.* (2021) worked on the predictions of restoring forces but with a main focus of the heave motion of the rectangular structure. It is important to further expand this work to put into account the surge motion of the rectangular barge and at the same time relating added mass, damping coefficients and the wave characters.

In the recent past there has been rising demands for energy globally and Cardoso *et al* (2021) has pointed out that the renewable sources of energy are becoming more accessible, where the ocean is the best source from its wind energy. In this regard, there is need for general stability of structures both in the production and transportation of offshore wind turbines that are floating freely. For the success of such structures, an adequate numerical analysis on the magnitude of wave loading which are subjected to them should be analyzed. The data from this analysis and especially those of wave restoring forces are of importance in analyzing the general stability of such structures. With considerations on the interactions of offshore structures with waves and the creation of loads around them, offshore engineers have been doing extensive researches to model better and efficient structures that can maintain high workability amidst harsh sea conditions. This study therefore adds value to this field by analyzing the restoring forces of a rectangular barge that is oscillating at zero forward speed. The results obtained is of importance in modelling of structures that can maintain high restorative forces.

2.2 Dispersion Relation of Ocean Waves

According to the frequency dispersion of water waves, waves of various wavelengths pass through the water's surface at various stages. Then, waves that go through the water's surface might be referred to as water waves. The restoring forces of surface tension and gravity operate upon these waves. Water particles on free surfaces are referred to as dispersed in a dispersive medium under these circumstances.

The dispersion relation is governed by the Airy wave theory. The mechanism of swell wave propagation on surfaces of homogeneous fluids is modelled using a linear theory when waves of small amplitude are assumed. This allows for the assumption of sinusoidal wave profiles without leaving out generality (Boccia *et al*., 2015).

The standard expression relating dispersion of ocean waves gives a relation between the frequency ω, gravitational acceleration g, water depth h and wave number k. The group velocity and phase velocities of waves in a given medium can be calculated with the use of a dispersion relation, which can be expressed as,

$$\omega^2 = gk \tanh kh \tag{2.1}$$

Dividing both sides of equation (2.1) with g gives,

$$\frac{\omega^2}{g} = k \tanh kh \tag{2.2}$$

Taking the coefficients of h and multiplying it to each part of the equation (2.2) above gives,

$$\omega^2 \frac{h}{g} = kh \tanh kh \tag{2.3}$$

Now finding the square root of both sides of equation (2.3) gives,

$$\omega\sqrt{\frac{h}{g}} = \sqrt{kh\tanh kh} \tag{2.4}$$

The linear dispersion relation is represented by equation (2.1). The dispersive nature of the waves is expressed by giving ω as a function of k. This relationship makes it clear that waves with different wavelengths often move at different rates, and when multiple waves are present, the longer-period waves will move more quickly.

The speeds at which the wave crest moves are indicated by the phase speeds of swell waves, which are,

$$c = \frac{\omega}{k} \tag{2.5}$$

$$c = \sqrt{\frac{g}{k}\tanh(hk)} \tag{2.6}$$

The aforementioned equation shows that wave height has no bearing on wave speed, C. Because waves of different lengths propagate at different speeds and finally scatter away or disperse, waves with C as a function of wave number k are called dispersive waves. Because the dispersive nature is indicated through the relation of ω as a function of k, a relation like that of equation (2.1) is a dispersion relation.

2.3 Boundary Conditions to be Satisfied

The availability of specific circumstances has allowed fluid mechanics researchers to successfully complete the majority of their studies on how waves affect offshore constructions. Boundary conditions are necessary for the fluid domain to be subjected to certain conditions in order for fluid motion to occur (Stepanyants & Sturova, 2021). In general, fluid dynamics researchers have developed the mathematical formulation of velocity potential with boundary conditions, which is an excellent tool for studying hydrodynamic loads. For this study, the following linearized boundary conditions were satisfied:

2.3.1 Bottom Boundary Condition

At the seabed, the velocity along the z-direction is zero in accordance with the Bottom Boundary Condition (BBC). This is because water cannot pass through the bottom of the channel, hence the vertical velocity of the water particles at the sea bottom must always be zero. The fluid's velocity field will have zero divergence since its density is constant throughout the medium. It implies from Falnes and Kurniawan (2020) that,

$$\nabla . V = 0 \tag{2.7}$$

Since the flow is irrotational and the curl of the fluid velocity is zero, curl k , then there exists a velocity potential $\phi(x, y, z)$ such that the respective velocity components in the (x, y, z) planes are defined as,

$$\frac{\partial \phi}{\partial x} = u ,$$

$$\frac{\partial \phi}{\partial y} = v ,$$

$$\frac{\partial \phi}{\partial z} = w \tag{2.8}$$

Since there existed a velocity potential, ϕ , equation (2.7) can be substituted into (2.8) to give the Laplace equation.

$$(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k).(\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k) = 0 \tag{*}$$

And from Zhou *et al.* (2021), the Laplace equation is representable as,

$$\nabla^2 \phi = 0 \tag{2.9}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{2.10}$$

Boundary conditions must be satisfied at the bottom and at the free surface. Consequently, the following are the boundary conditions for this case:

The velocity components at the seabed, $z = -h$, must go to zero,

$$w = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = -h \tag{2.11}$$

Equation (2.11) is the Bottom Boundary Condition.

2.3.2 Dynamic Free Surface Boundary Condition

Given that the fluid being studied is irrotational in nature and that the free surface pressure is assumed to be equal to the atmospheric pressure, Bernoulli's equation, as presented by Gnitko *et al.* (2021), is relevant,

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + \frac{p}{\rho} + gz = 0 \tag{2.12}$$

and,

$$p = 0 \text{ at } z = \eta \tag{2.13}$$

The non-linear terms $\left(u^2+v^2+w^2\right)$ are neglected for waves of small amplitudes and therefore the general linearized Bernoulli equation is given as,

$$\frac{\partial\phi}{\partial t}+\frac{p}{\rho}+gz=0 \tag{2.14}$$

Substituting equation (2.13) to (2.14) gives,

$$\left(\frac{\partial\phi}{\partial t}\right)_{z=0}+g\eta=0 \text{ and } z=\eta\left(x,y,t\right) \tag{2.15}$$

Where $z=\eta(x,y,t)$ is a representation of surface displacement of the fluid from its mean level.

Equation (2.15) is the Dynamic Free Surface Boundary Condition.

2.3.3 Kinematic Free Surface Boundary Condition

At the free surface, there is a kinematic free surface boundary condition (KFSBC) that mandates that the fluid particles never escape the surface. This suggests that the fluid particle follows the fluid surface and the fluid surface follows the fluid particle (Yong-Can *et al.*, 2022). It shows that if the fluid particle travels along the tangent line, it will stay on the free surface. The free surface must follow the particle since it will leave it if it goes in a normal direction. As a result, as long as the wave does not break, the fluid particle remains on the free surface continually.

Let $F\left(x,y,\mathrm{z}\right)$ (2.16)

$$\frac{DF}{Dt}=\left(\frac{\partial F}{\partial t}+u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}-z\right)_{z=0}=0 \tag{2.17}$$

$$\frac{\partial\eta}{\partial t}+\frac{\partial\eta}{\partial x}\left(\frac{\partial\phi}{\partial x}\right)_{z=\eta}+\frac{\partial\eta}{\partial y}\left(\frac{\partial\phi}{\partial y}\right)_{z=\eta}=\left(\frac{\partial\phi}{\partial z}\right)_{z=\eta} \tag{2.18}$$

From equation (2.18), $\left[\frac{\partial\eta}{\partial x}\left(\frac{\partial\phi}{\partial x}\right)_{z=\eta}\right]$ and $\left[\frac{\partial\eta}{\partial y}\left(\frac{\partial\phi}{\partial y}\right)_{z=\eta}\right]$ are small for small-amplitude waves. Equation (2.18) therefore decomposes to,

$$\frac{\partial\eta}{\partial t}=\left(\frac{\partial\phi}{\partial z}\right)_{z=\eta} \tag{2.19}$$

$z=\eta$ can then be evaluated at $z=0$ rather than at the free surface.

$$\frac{\partial\eta}{\partial t}=\left(\frac{\partial\phi}{\partial z}\right)_{z=0} \tag{2.20}$$

Equation (2.20) is the Kinematic Free Surface Boundary Condition.

2.3.4 Radiation Condition

At distances from the body, the surface waves must vanish along with the velocity potential in order to meet this requirement. As R→∞, it suggests that |∇ϕ|→0. According to this condition, surface waves deteriorate across distances from their source.

$$\lim_{R\to\infty}\sqrt{R}\left(\frac{\partial\phi}{\partial\eta}-ik\phi\right)=0 \tag{2.21}$$

2.4 Body Response in Ocean Waves

Naval engineers have always been interested in the impacts that floating or submerged vessels experience when waves are present. The ocean environment has long been characterized as a random process that results in random reactions from the water and structures that operate there (Khan *et al.*, 2022). It is necessary to make accurate hydrodynamic load calculations while designing these offshore constructions. When studying fluid motions and how they affect both fixed and floating marine structures, hydrodynamic loads are crucial, and understanding them is useful. It is important for structures to be designed to withstand the harsh conditions of the sea. The amplitudes of the waves are assumed to be sufficiently small in this work to support linearization and allow for the superpositions of particular wave-body interactions when regular waves are present (Newman, 2018). As a result, the floating body experiences planar progressive waves with an amplitude of Q and a direction of ϕ. The body is thus compelled to move in its six degrees of freedom in reaction to the waves. These six degrees of freedom can be classified as rotational (roll, yow, and pitch) or translational (surge, heave, and sway).

The body's oscillatory motion is harmonic and taken to be of the same frequency ω as that of the incoming wave and the corresponding velocities from the oscillation is represented as,

$$\zeta_j(t)=\mathrm{Re}[\mathrm{i}\,\omega q_j e^{-iwt}] \tag{2.22}$$

q_j is the complex body motion's amplitude. The resulting motion will be proportionately small due to the assumption that the body is stable and incident waves exhibit smaller amplitudes. Li *et al*. (2022) expresses the general velocity potential as,

$$\phi(x,y,z,t)=\mathrm{Re}\left\{(\phi(x,y,z)\,e^{-iwt})\right\} \tag{2.23}$$

Re denotes the real part of the complex quantity. The induced body motion's period and the incident wave's period are represented by T and derived from $\omega=\frac{2\pi}{T}$. Endo (1987) asserted that the resulting motions of these structures are relatively minimal in accordance with linear theory, and this assertion is supported by the assumption that the incident waves have appropriately modest amplitudes. The general velocity potential of any floating structure or vessel can be expressed using the linear potential theory as the total of the potentials resulting from diffraction, radiation potential, and undisturbed incoming waves in accordance with the six body motions (Faltinsen, 1974). That is,

$$\phi = \phi_I + \phi_D + \sum_{j=1}^{6} \phi_j \tag{2.24}$$

In the equation (2.24) above, ϕ_j has been used to represent the solution to the corresponding radiation potential. ϕ_I is a representation of the solution of the incident velocity potential and ϕ_D are the diffracted velocity potentials.

The complex velocity potential ϕ of the oscillating body is expressible as,

$$\phi = -i\omega \sum_{j=0}^{7} \phi_j q_j \tag{2.25}$$

Here, and from wave theory, the incident velocity potential of the incident waves of small amplitudes is expressible as,

$$\phi(x,y,z) = \frac{g}{\omega^2} . \frac{\cosh k(z+h)}{\cosh(kh)} . \exp\{ik(x\cos(x) + y\sin(x)\} \tag{2.26}$$

where, k is a representation of wave number and,

$$\frac{\omega^2}{g} = k\tanh(kh) \tag{2.27}$$

With h being the water depth.

Each component of the complex potential function in equation (2.25) satisfies the Laplace equation $\nabla^2\phi = 0$ for $j = 0,1, \ldots \ldots .7$

The free surface boundary condition, which is a combination of DFSBC and the KFSBC, is expressed as;

$$\frac{\partial \phi_j}{\partial z}(x,y,0) - \frac{\omega^2}{g}\phi_j(x,y,0) = 0 \qquad j = 0,1, \ldots .7 \tag{2.28}$$

And the sea bottom boundary condition is expressed as,

$$\frac{\partial \phi_j}{\partial z}(x,y,-h) = 0 \qquad j = 0,1 \ldots \ldots 7 \tag{2.29}$$

It is preferable to prescribe boundary conditions on the body surface, on the sea bottom and also on the free surface. Since the body's motion is assumed to be of small amplitudes, then the kinematic boundary conditions on the immersed surface on the mean position is,

$$\frac{\partial \phi_j}{\partial n} = i\omega n_j \qquad j = 1, 2, ...6 \tag{2.30}$$

$$\frac{\partial \phi_j}{\partial n} = i\omega(\mathrm{B} \times n) \tag{2.31}$$

on the surface of the body S_B.

In (2.31), n is taken to be the unit normal to the body surface vector, and is directed to the body and B is taken to be the position vector (x, y, z).

The boundary conditions on the body surface are,

$$\frac{\partial \phi_Q}{\partial n} = 0 \text{ on } S_B \tag{2.32}$$

The operator $\frac{\partial}{\partial \phi}$ is used to indicate derivatives in the outward normal direction from the body's surface. The radiation condition ϕ_j is determined by expressing the boundary value problem in terms of integrals equations and then solving it. For this to be possible, the Green's theorem is employed. At a point, let's say $P(x, y, z)$ on the fluid domain, the velocity potential due to the pulsating source is expressible in terms of the surface distributions of sources by Faltinsen (1974) as,

$$\phi_j(x, y, z) = \iint_s \delta_j(\xi, \eta, \zeta) G(x, y, z; \xi, \eta, \zeta) ds \tag{2.33}$$

From equation (2.33) above, $\delta_j\,(\xi, \eta, \zeta)$ is a depiction of the undiscovered source density functions associated with the j^{th} oscillation mode and $G(x, y, z; \xi, \eta, \zeta)$ is the Green's function for the problem and which from (Wehausen & Laitone, 1960) can be represented in two forms. That is,

$$G = \frac{1}{r} + \frac{1}{r_1} + PV\int_0^\infty \frac{2(k+v)\exp(-\mathrm{kh})\cosh k(h+b)}{k\sinh(kh) - v\cosh(kh)} \cosh k(y+h) J_0(kR) dk \tag{2.34}$$

And its series form which is expressed as,

$$G = 2\pi \frac{v^2 - k^2}{hk^2 - hv^2 + v} \cosh k(y+h) \cosh k(\eta + h) \times \left[Y_0(kR) + iJ_0(kR)\right] \tag{2.35}$$

where,

$$R = [(x-\xi)^2 + (y-\eta)^2]^{\frac{1}{2}} \tag{2.36}$$

$$r = [(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{\frac{1}{2}} \tag{2.37}$$

$$r_1 = [(x-\xi)^2 + (y-\eta)^2 + (z+2h+\zeta)^2]^{\frac{1}{2}} \tag{2.38}$$

$$v = \frac{\omega^2}{g} = k.\tanh(kh) \quad (2.39)$$

Where k is the dispersion relation equation's real positive root and PV is the integral's Principal Value, and it possesses singularities. J_0 is the zero order Bessel function of the first kind while Y_0 is the second order second kind Bessel function of zero order. Wehausen and Laitone (1960) developed the series form of the Greens function expressed in equation (2.35) in response to numerous hydrodynamics researchers who have noted that the integral form of the Greens function represented in equation (2.34) is difficult to use.

Applying the Green's second identity below,

$$\iiint_v (\phi_j \nabla^2 G - G \nabla^2 \phi_j)\,\mathrm{dv} = \iint_s \phi_j \left(\frac{\partial G}{\partial n} - G \frac{\partial \phi_j}{\partial n} \right) ds = 0 \quad (2.40)$$

to equation (2.35) yields,

$$-2\pi \delta_j \left(x, y, z\right) + \iint_s \delta_j \left(\xi, \eta, \zeta\right) \frac{\partial G}{\partial n} \left(x, y, z; \xi, \eta, \zeta\right) dv = n_j \quad (2.41)$$

The second type of potential, which includes the radiation potential on the body's surface, is represented by the Fredholm equation (2.41). It's solved by simulating the body with a huge number of plane quadrilateral elements, each with a constant source density. In the unknown values of the source density on the elements, this changes the integral equation into a sequence of linear algebraic equations (Truong, 2022). Equation (2.41) may not always produce the correct results in circumstances of irregular frequencies. The frequency irregularity, on the other hand, will not be present if the body has no forward speed.

After determining the source density δ_J, the normalized potentials ϕ_j in equation (2.33) can be calculated using an integration approach that involves integrating sources over quadrilaterals. The Bernoulli equation will be used to calculate the pressure, as well as the added mass and damping factors. This makes analyzing restorative forces even easier.

2.5 Numerical Methods

The solution of hydrodynamics problems requires the employment of different numerical methods. Among several numerical methods which exists are the strip method, the panel method, the Finite Element Method and the Boundary Element Method. Marine hydrodynamics encompasses fluid mechanics applications that are relevant to ships, offshore structures, and other sea-based structures (Deng *et al.*, 2022). Traditionally, these issues were represented empirically, but breakthroughs in theoretical knowledge and computational power have allowed much of this work to be transferred to numerical solutions.

Panel methods are a flexible framework for solving potential-flow problems in marine hydrodynamics numerically. It is an analysis method which can be used in arriving at approximate solutions for the forces acting on specific object in a flow and its main goal is determining the surface velocity and applying the Bernoulli equation to find the local pressure distributions (Kim & Nam, 2022). The pressure gotten is integrated over the surface to get the forces generated by the fluid flow. The capacity to show outside flows and bodies with arbitrary geometrical forms in three dimensions is one of this approach's main features. If the Green function that meets the extra constraints is known, it is simple to impose additional boundary conditions such as the free surface using the panel technique. Furthermore, fast advancements in computer hardware have made panel approaches more accessible in a variety of situations, from desktops to supercomputers.

Numerical solutions to the Laplace equation in the fluid domain was sought, subject to the proper boundary conditions, under assumptions of potential theory. Velocity potential was stated on the body, the fluid was then extended in all directions to infinity, and the solution specified at infinity in the simplest circumstances (Newman, 1992). The most effective numerical technique, given the fluid's unbounded nature, was spreading the sources and normal dipoles on the body surface. Lamb (1932) provides the rationale for this strategy. With the source intensity equal to the known normal velocity and the dipole moment equal to the unknown potential, the Greens theorem was employed directly in the potential formulation. The velocity potential of the body was then solved using a second Fredholm equation. Hess and Smith (1964) were the first to apply this method to three-dimensional bodies of any shape.

In general, panel methods are ideally adapted to the solution of three-dimensional hydrodynamic problems and are most effective in cases of structures with large domains such as the floating platforms because only the boundary of the domain will be discretized (Liu & Soares, 2022).

CHAPTER THREE

MATERIALS AND METHODS

3.1 Wave Characters of Ocean Waves

3.1.1 Mathematical Formulation

The fluid motion was three dimensional in the $O - xyz$ plane and waves travelled along the x direction. The z axis was the vertical axis that was measured upward from the undisturbed free surface. The $O - xy$ plane lied on the undisturbed free surface. The displacement of surface water was given by $z = \eta(x, y, t)$. The wave elevation was assumed to be equal to the displacement of free surface water. Water depth was taken to be the distance between the sea bed $z = -h$ and the still water level $z = 0$. The wavelength of the wave λ was assumed to be greater than the wave amplitude. Since the wave under study is periodic in nature, the wave period was taken as the time used by one wave to pass a particular point. This problem's solution was subject to particular boundary conditions.

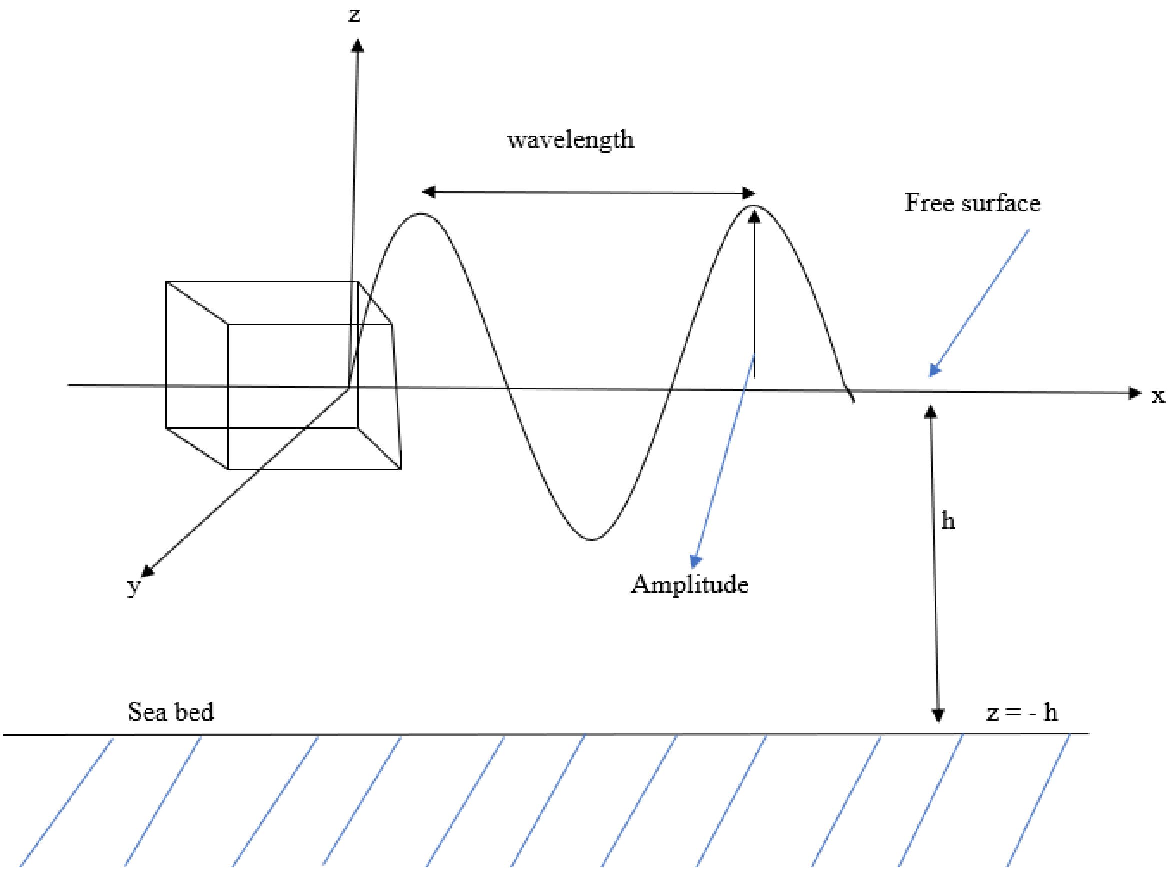

Figure 3.1: Schematic representation of outgoing waves from a source.

The figure above is a schematic diagram of the outgoing waves. It was taken that the incoming waves hit the structure and made it to oscillate. With its oscillation, the body then moved in different oscillatory and translatory motions and thus causing outgoing waves.

3.1.2 Governing Equations to be Satisfied

For accurate attainment of the velocity potential of the waves from the source, it was important to keep key considerations on the boundary conditions that governs the study. The following are the principles and the boundary conditions that guided the attainment and derivation of the velocity potential;

I. The flow under study was assumed to be irrotational and a velocity potential $\phi(x, y, z)$ existed.

II. The Laplace equation $\nabla^2\phi = 0$ was taken to be the governing equation and was satisfied by the velocity potential.

III. The dynamic free surface condition should exist. That is

$$\frac{\partial \phi}{\partial t} + g\eta = 0 \qquad (3.1)$$

IV. The kinematic free surface condition should exist. That is;

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \qquad (3.2)$$

And from equation (3.1) and equation (3.2), expression (3.3) was arrived at as;

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0 \ , \ z = 0 \qquad (3.3)$$

At a time when the velocity potential ϕ oscillates harmonically in time with a circular frequency ω, then equation (3.3) can be expressed as;

$$-\omega^2\phi + g\frac{\partial \phi}{\partial z} = 0 \qquad \text{at} \qquad z = 0 \qquad (3.4)$$

In these conditions, g is the gravity-induced acceleration.

3.1.3 Velocity Potential of the Outgoing Waves

Considering that the form below represents the elevation of the free surface waves;

$$\eta = qe^{i(k_1 x\cos\theta + k_2 y\sin\theta - \omega t)} \qquad (3.5)$$

The main goal is to identify the potential (ϕ) that meets each of the previously listed boundary conditions assuming that each function depends on one independent function and that the velocity potential is a representation of the product of functions. The separation of variables approach can thus be used to solve the Laplace equation in this respect.

Let the incident wave potential take the form given below;

$$\phi = f(z)e^{i(k_1 \text{x} \cos\theta + k_2 \text{y} \sin\theta - \omega t)} \tag{3.6}$$

In which f represents the restoring forces.

Differentiating the equation (3.6) twice to get the second order derivative gives;

$$\nabla^2\phi = \frac{\partial^2 f}{\partial z^2} - k^2 f(\text{z}) = 0 \tag{3.7}$$

In which,

$$k^2 = \sqrt{k_1^2 + k_2^2} \tag{3.8}$$

The general solution of equation (3.7) can then be given as;

$$z = Ne^{kz} + Je^{-kz} \tag{3.9}$$

Substituting equation (3.9) into equation (3.6) gives;

$$\phi = (\text{N}e^{kz} + Je^{-kz})(e^{i(k_1 x \cos\theta + k_2 y \sin\theta - \omega t)}) \tag{3.10}$$

Using the Laplace equation and the second order derivative of equation (3.10), the following results;

$$Ne^{-kh} - Je^{kh} = 0$$

$$Ne^{-kh} = Je^{kh} \tag{3.11}$$

Substituting equation (3.10) into equation (3.4) gives;

$$(\omega^2 - gk)\,\text{N} + (\omega^2 + gk)\,\text{J} = 0 \tag{3.12}$$

Writing equations (3.11) and (3.12) in matrix form gives;

$$\begin{vmatrix} e^{-kh} & -e^{kh} \\ (\omega^2 - gk) & (\omega^2 + gk) \end{vmatrix} = 0 \tag{3.13}$$

Solving equation (3.13) and making ω^2 the subject gives;

$$e^{-kh}(\omega^2 + gk) + e^{kh}(\omega^2 - gk) = 0$$

$$\omega^2 e^{-kh} + gke^{-kh} + \omega^2 e^{kh} - gke^{kh} = 0 \tag{3.14}$$

Collecting the like terms give;

$$\omega^2\left(e^{kh} + e^{-kh}\right) - gk\left(e^{kh} - e^{-kh}\right) = 0$$

Therefore,

$$\omega^2 = gk \tanh kh \tag{3.15}$$

Expressing the wave celerity with the expression below;

$$c = \frac{\omega}{k} \tag{3.16}$$

And the wave frequency as;

$$\omega = \sqrt{gk \tanh kh} \tag{**}$$

$$\omega^2 = gk \tanh kh \tag{3.17}$$

Divide both sides of equation (3.17) with g

$$\frac{\omega^2}{g} = k \tanh kh \tag{3.18}$$

Multiplying both sides of equation (3.18) by the coefficients of h gives;

$$\omega^2 \frac{h}{g} = kh \tanh kh \tag{3.19}$$

and finding the square root of both sides of equation (3.19) gives;

$$\omega\sqrt{\frac{h}{g}} = \sqrt{kh \tanh kh} \tag{3.20}$$

Then;

$$c = \sqrt{\frac{g}{k} \tanh kh} \tag{3.21}$$

Letting $\quad Ne^{-kh} = Je^{kh}$ (3.22)

Applying equation (3.13) into equation (3.1) ends up in obtaining the equation below;

$$\phi = \frac{1}{2} H \left\{ e^{k(h+z)} + e^{-k(h+z)} \right\} \left(e^{i(kx\cos\theta + ky\sin\theta - \omega t)} \right) \tag{3.23}$$

And by the definition of hyperbolic function, that is;

$$\cosh k(\mathrm{h+z}) = \frac{1}{2}\left(e^{k(h+z)} + e^{-k(h+z)} \right) \tag{***}$$

Substituting equation (***) to equation (3.23) gives;

$$\phi = \left(\mathrm{H} \cosh k\left(h+z\right)\right) e^{i(kx\cos\theta + ky\sin\theta - \omega t)} \tag{3.24}$$

However, equation (3.1) can be expressed as equation (3.25) below

$$\left(\frac{\partial \phi}{\partial t}\right)_{z=0} = -g\eta \tag{3.25}$$

Performing a differentiation on equation (3.24) with respect to time gives;

$$\left(\frac{\partial \phi}{\partial t}\right) = -i\omega H \cosh(kh)\, \mathrm{e}^{i(kx\cos\theta + ky\sin\theta - \omega t)} \tag{3.26}$$

Applying equation (3.4) to equation (3.26) above gives;

$$\eta = -\frac{1}{g}\left\{-i\omega H \cosh(kh)\,\mathrm{e}^{i(kx\cos\theta + ky\sin\theta - \omega t)}\right\} \tag{3.27}$$

Therefore;

$$\eta = \frac{-1}{g}\left(\frac{\partial \phi}{\partial t}\right) \tag{3.28}$$

Expressing $q = -\frac{i\omega}{g}\mathrm{H}\cosh kh$ (3.29)

and reducing equation (3.28) to the form below;

$$\eta = qe^{i(kx\cos\theta + ky\sin\theta - \omega t)} \tag{3.30}$$

Getting back to equation (3.15) and expressing it in the form below;

$$\frac{\omega^2}{g} = k\tanh kh \tag{3.31}$$

And since $\tanh kh = \frac{\sinh kh}{\cosh kh}$ then equation (3.15) can be expressed as;

$$\frac{\omega^2}{g}\cosh kh = k\sinh kh \tag{3.32}$$

From (3.29),

$$H = -i\frac{qg}{\omega\cosh kh} \tag{3.33}$$

And from equation (3.32);

$$\cosh kh = \frac{g}{\omega^2}k\sinh kh \tag{3.34}$$

Substituting the equation (3.34) to (3.33) gives;

$$H = -i\frac{q\omega}{k}\frac{1}{\sinh kh} \tag{3.35}$$

Substituting equation (3.35) into equation (3.24) then the velocity potential due to the incident waves is given in the form below;

$$\phi = -i\frac{q\omega}{k}\frac{\cosh k(\mathrm{h}+\mathrm{y})}{\sinh kh}e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \tag{3.36}$$

And since scientifically, the incoming wave was generated from a source, then the velocity potential in equation (3.36) is taken to predict the outgoing velocity potential generated from a source that is hit and made to oscillate by this incident wave.

And from Euler's formula for ordinary trigonometric functions expression (3.09) is true;

$$e^{i\alpha t} = \cos\alpha t + i\sin\alpha t \tag{3.37}$$

Using equation (3.37) into equation (3.36) gives;

$$e^{i\alpha t} = \left[\cos \mathrm{kx} \cos\theta + kz\sin\theta - \omega \mathrm{t} + \mathrm{i}\sin(\mathrm{kx}\cos\theta + \mathrm{kz}\sin\theta - \omega t)\right]$$

$$\phi = \frac{-iq\omega}{k}\frac{\cosh k(h+z)}{\sinh kh}\left[\cos\left(kx\cos\theta + ky\sin\theta - \omega t\right) + i\sin\left(kx\cos\theta + ky\sin\theta - \omega t\right)\right] \qquad (3.38)$$

Opening equation (3.38) gives the real and imaginary parts. The real part of the equation is presented as;

$$\phi = \left(-i\frac{q\omega}{k}\frac{\cosh k(h+z)}{\sinh kh}\right)\left[\mathrm{i}\sin(\mathrm{kx}\cos\theta + \mathrm{ky}\sin\theta - \omega t)\right] \qquad (3.39)$$

$$\phi = \frac{q\omega}{k}\frac{\cosh k(h+z)}{\sinh kh}\sin\left(kx\cos\theta + ky\sin\theta - \omega t\right) \qquad (3.40)$$

Equation (3.40) is the velocity potential satisfying the boundary conditions mentioned earlier.

3.1.4 Wave Elevation

Equation (3.40) is differentiated with respect to time to obtain the incident wave elevation. That is;

$$\frac{\partial\phi}{\partial t} = \frac{-q\omega^2\cosh k(h+z)}{k\sinh kh}\cos(kx\cos\theta + ky\sin\theta - \omega t) \qquad (3.41)$$

Applying equation (3.25) to equation (3.41) gives;

$$\frac{\partial\phi}{\partial t} = \frac{q\omega^2\cosh k(h+z)}{k\sinh kh}\cos(kx\cos\theta + ky\sin\theta - \omega t) \qquad (3.42)$$

The outgoing wave elevation is predicted using equation (3.42), which is the expression for the incident wave elevation. Since it is utilized to calculate the vertical motion of water structures from the undisturbed free surface, wave elevation is a crucial wave characteristic. When coastal engineers forecast potential ship damage from slamming, this wave character is crucial.

3.1.5 Wave Velocity

In getting the wave velocity, equation (3.40) is differentiated partially along each axis. That is;

$$u = \frac{q\omega\cos\theta\cosh k(h+z)}{\sinh kh}\cos(kx\cos\theta + ky\sin\theta - \omega t)$$

$$v = \frac{q\omega\sin\theta\cosh k(h+z)}{\sinh kh}\cos(kx\cos\theta + ky\sin\theta - \omega t) \qquad (3.43)$$

$$w = \frac{q\omega\sinh k(h+z)}{\sinh kh}\sin(kx\cos\theta + ky\sin\theta - \omega t)$$

The fluid's local velocities at a distance above the impermeable bottom are expressed by the equations above. The exponential decays of the velocity component magnitudes with increasing depth below the free surface are represented by the hyperbolic functions in equation (3.43).

3.1.6 Wave Acceleration

Equation (3.43) will be differentiated with respect to time. That is;

$$u_t = \frac{q\omega^2 \cos\theta \cosh k(h+z)}{\sinh kh} \sin(kx\cos\theta + ky\sin\theta - \omega t)$$

$$v_t = \frac{q\omega^2 \sin\theta \cosh k(h+z)}{\sinh kh} \sin(kx\cos\theta + ky\sin\theta - \omega t) \qquad (3.44)$$

$$w_t = -\frac{q\omega^2 \sinh k(h+z)}{\sinh kh} \cos(kx\cos\theta + ky\sin\theta - \omega t)$$

The negative sign shows that the acceleration of the wave is changing with respect to the origin.

3.2 Hydrodynamic Forces acting on a Floating Structure

The body under consideration in this study was a stationary floating body which was situated in calm water. It was assumed that the body was acted upon by incoming waves making it to oscillate and produce outgoing waves. In the solution of the wave forces acting on this body, the Laplace equation $\nabla^2\emptyset = 0$ was taken as the governing equation and since the problem is a linear boundary value problem, it was important to put linear theory into use.

3.2.1 Response of a Floating Structure in Regular Waves

Offshore structures respond differently to the action of waves. The response of these structures to waves depends on the modelling of the structure and on the extent at which the knowledge of hydrodynamic loads was utilized in the design process. Hydrodynamic loads associated with structures oscillating in the sea are the added mass, damping and restoring forces and moments. Much has been done regarding the analysis of added mass and damping coefficients of structures of different shapes. Typically, an assumption is taken that the floating structures are rigid bodies whose hydrodynamic restoring forces and moments are all due to the infinitesimal changes of the translatory and rotational displacements (Khair Al-Solihat & Nahon, 2015).

Studies on restoring forces have remained unworked on for some time. This gives this research work a lee way of looking into this problem and predicting these wave loads. For this to be achievable, it was important that added mass and damping coefficients were derived for they play a role in the analysis of restoring forces.

3.2.1.1 Added Mass and Damping Coefficients

Added mass and damping are among hydrodynamic loads that are damped around floating, fixed and moving structures in the sea. Research has it that these hydrodynamic loads are not easy to analyze but their coefficients can be analyzed. These hydrodynamic loads can be derived from the generalized form of the Bernoulli equation with the linear terms retained given as,

$$p = -\rho \frac{\partial \phi}{\partial t} \tag{3.45}$$

Since the fluid interacts with the structure in the sea, it does exert some pressure on this structure and makes it to oscillate and generate radiation forces and moments. These forces can be analyzed by performing an integration of the Bernoulli equation over the wetted surface of the body. This can be expressed as,

$$\begin{pmatrix} F \\ M \end{pmatrix} = \iint_{S_B} p \begin{pmatrix} \vec{n} \\ \vec{r} \times \vec{n} \end{pmatrix} dS \tag{3.46}$$

Where $\vec{n}$ is the unit normal vector acting on the body surface while $\vec{r}$ is the position vector (x, y, z). On substituting equation (3.45) into (3.46),

$$\begin{pmatrix} F \\ M \end{pmatrix} = -\rho \iint_{S_B} \frac{\partial \phi}{\partial t} \begin{pmatrix} \vec{n} \\ \vec{r} \times \vec{n} \end{pmatrix} dS \tag{3.47}$$

Equation (3.47) has both the radiated force component and the radiated moment component. The point of interest with respect to hydrodynamic loading is the radiated force component which is represented as,

$$F_R = -\rho \iint_{S_B} \frac{\partial \phi}{\partial t} \vec{n} dS \tag{3.48}$$

Considering that the waves under study are small amplitude waves, then the complex amplitudes are taken to be independent parameters. This then follows that,

$$\frac{\partial \phi_j}{\partial \eta} = -i\omega n_j \tag{3.49}$$

On making n_j the subject, then equation (3.49) becomes,

$$n_j = -\frac{1}{i\omega} \cdot \frac{\partial \phi_j}{\partial \eta} \tag{3.50}$$

Substituting equation (3.50) to (3.48) gives,

$$F_R = -\frac{i\rho}{\omega} \iint_{S_B} \frac{\partial \phi}{\partial t} \frac{\partial \phi_j}{\partial \eta} dS \tag{3.51}$$

From Newman (2018), the total velocity potential acting on the body is represented as,

$$\phi(x,y,z) = -i\omega\sum_{j=1}^{6}\xi_j\phi_j + A(\phi_I + \phi_D) \tag{3.52}$$

Which can further be represented as a time harmonic factor as,

$$\phi(x,y,z,t) = \mathrm{Re}\left[-i\omega\sum_{j=1}^{6}\xi_j\phi_j e^{i\omega t} + A(\phi_I + \phi_D)\,\mathrm{e}^{i\omega t}\right] \tag{3.53}$$

Differentiating the first term of the right-hand side of equation (3.53) gives,

$$\frac{\partial\phi}{\partial t} = \mathrm{Re}\sum_{j=1}^{6}\omega^2\xi_j\phi_j e^{i\omega t} \tag{3.54}$$

Substituting equation (3.54) to (3.51) we get,

$$F_R = \rho\,\mathrm{Re}\sum_{j=1}^{6} i\omega\xi_j e^{i\omega t}\iint_{S_B}\phi_j\frac{\partial\phi_j}{\partial\eta}dS \tag{3.55}$$

Equation (3.55) above is a representation of the radiated force component. This is a complex component which is also representable as a sum of the product of added mass and acceleration and the product of damping and velocity and is expressible as,

$$\rho\,\mathrm{Re}\sum_{j=1}^{6} i\omega\xi_j e^{i\omega t}\iint_{S_B}\phi_j\frac{\partial\phi_j}{\partial\eta}dS = \ddot{\xi}_j A_{ij} + \dot{\xi}_j B_{ij} \tag{3.56}$$

Simplifying equation (3.56) further gives,

$$\rho\iint_{S_B}\phi_i n_j = A_{ij} + \frac{B_{ij}}{\omega} \tag{3.57}$$

Equation (3.57) then takes the form,

$$f_{ij} = \rho\iint_{S_B}\phi_j\frac{\partial\phi_i}{\partial n}dS \tag{3.58}$$

Equation (3.57) is a complex form and the force component is expressible as,

$$f_{ij} = \omega^2 A_{ij} + i\omega B_{ij} \tag{3.59}$$

From equation (3.59), the added mass coefficient and the damping coefficient can be derived. That is,

$$A_{ij} = \frac{1}{\omega^2}\mathrm{Re}\left|f_{ij}\right| \tag{3.60}$$

Which is the real part and is expressible from equation (3.58) as,

$$A_{ij} = \frac{1}{\omega^2}\mathrm{Re}\sum_{j=1}^{6}\sum_{j=1}^{6}\rho\iint_{S_B}\frac{\partial\phi_i}{\partial n}\phi_j dS \tag{3.61}$$

The damping coefficient is representable as,

$$B_{ij} = \frac{1}{\omega} \mathrm{Im} \left| f_{ij} \right| \tag{3.62}$$

Which is then representable as,

$$B_{ij} = \frac{1}{\omega} \mathrm{Im} \sum_{j=1}^{6} \sum_{j=1}^{6} \rho \iint_{S_B} \frac{\partial \phi_i}{\partial n} \phi_j dS \tag{3.63}$$

3.2.1.2 Restoring Force Coefficients

From Newton's third law of motion, for every action, there should be an equal and opposite reaction. Therefore, for every exiting force there is an opposite and equal restoring force that tries to bring a body back to its equilibrium position. The study of restoring forces is essential in modelling structures with high stability. Coefficients of restoring forces were analyzed from relating added mass and damping coefficients to wave acceleration and velocity. This brings the correlation that exist between wave characters and added mass and damping coefficients in the attainment of coefficients of restoring forces.

3.3 Body Response in various degrees of motion

Heave motion is the up and down movement of a ship when acted upon by waves. The coefficient of the heave motion is taken to be η_3.

Taking the wave amplitude in the heave oscillation to be

$$\vec{\eta}_3 = \eta_0 e^{i\omega t} . \vec{k} \tag{3.64}$$

and for a sinusoidal motion,

$$\begin{aligned} \vec{\dot{\eta}}_3 &= i\omega.\eta_0 e^{i\omega t} . \vec{k} \\ i\omega \vec{\eta}_3 &= \dot{\eta}_3 \vec{k} \end{aligned} \tag{3.65}$$

Differentiating equation (3.65) once and substituting equation (3.64) into (3.65) gives;

$$\vec{\ddot{\eta}}_3 = -\omega^2 \eta_0 e^{i\omega t} \vec{k} \tag{3.66}$$

A function u_n , which is the body velocity and which is normal to the body, is introduced. Then,

$$\begin{aligned} u_n &= \vec{n} . \overrightarrow{\dot{\eta}_3} \\ i\omega \eta_0 e^{iwt} \vec{k} . \vec{n} &= \dot{\eta} \left(\vec{k} . \vec{n} \right) \end{aligned} \tag{3.67}$$

In the fluid;

$$\vec{V} = \nabla \phi_3 \tag{3.68}$$

And the normal velocity on the surface is given as;

$$\vec{V}_n = \frac{\partial \phi_3}{\partial n} \tag{3.69}$$

On giving a definition such that;

$$\phi_3\left(\vec{r},t\right) = i\omega\eta_0 e^{iwt}\phi_3\left(\vec{r}\right) \tag{3.70}$$

Where $i\omega\eta_0 e^{iwt}$ is a time dependent function and $\phi_3(\vec{r})$ is a space dependent function.

With considerations that the normal body velocity equals the normal velocity of the fluid, $u_n = v_n$ then the normal velocity of the fluid is expressed as;

$$\begin{aligned} V_n &= \vec{n}.\nabla\phi_3 \\ \dot{\eta}_3\left(\vec{n}.\nabla\phi_3\right) &= \dot{\eta}_3 \frac{\partial \phi_3}{\partial \eta} \end{aligned} \tag{3.71}$$

And the normal velocity on the body surface is expressed as;

$$u_n = \dot{\eta}_3\left(\vec{k}.\vec{n}\right) \tag{3.72}$$

Since the normal velocities in the body surface equals the normal velocities in the fluid then;

$$\begin{aligned} u_n &= V_n \\ \dot{\eta}_3\left(\vec{k}.\vec{n}\right) &= \dot{\eta}_3 \frac{\partial \phi_3}{\partial \eta} \end{aligned} \tag{3.73}$$

And $\frac{\partial \phi_3}{\partial n} = \vec{n}.\vec{k}$ on the body surface.

Letting $\vec{k} = \vec{e}_3$ along the z axis gives;

$$\frac{\partial \phi}{\partial n} = \vec{n}.\vec{e}_3\Big|_{S_{Body}} \tag{3.74}$$

This is a representation of the heave motion.

The same is repeated on all the 6 degrees motions to give the following general representation;

$$\frac{\partial \phi_j}{\partial n} = \vec{n}.\vec{e}_j\Big|_{S_{Body}} \tag{3.75}$$

j is a representation of direction of motion of the body in its six degrees of freedom.

From these boundary conditions, the coefficients of restoring forces will be derived.

The radiated force equation (3.59) can be rewritten as,

$$\begin{aligned} F_{ij} &= \left(a+ib\right)\ddot{\eta}_j \\ a\ddot{\eta}_j + ib\left(i\omega\dot{\eta}_j\right) &= a\ddot{\eta}_j - \omega b\dot{\eta}_j \end{aligned} \tag{3.76}$$

Where;

$$F_{ij} = a\ddot{\eta}_j - \omega b\dot{\eta}_j \tag{3.77}$$

i is a representation of the force direction and j is a representation of the motion direction. Then by definition and since the force should act in the opposite direction,

$$F_{ij} = -A_{ij}\ddot{\eta}_j - B_{ij}\dot{\eta}_j \tag{3.78}$$

The equation above is a representation of restoring forces. Therefore, restoring forces is a sum of the added mass and the damping effect. Where in our equation,

$A_{ij} = -a$ and is a representation of added mass which is the force component that is proportional to the acceleration of the body.

$B_{ij} = \omega b$ and is a representation of damping which represents the force component that is proportional to the velocity of the body.

From equation (3.61) and (3.63) the coefficients of added mass and damping are as follows;

$$A_{ij} = \frac{1}{\omega^2} \operatorname{Re} \sum_{i=1}^{6}\sum_{j=1}^{6} \rho \iint_{S_B} \frac{\partial \phi_i}{\partial n}\phi_j ds \tag{3.79}$$

$$B_{ij} = \frac{1}{\omega} \operatorname{Im} \sum_{i=1}^{6}\sum_{j=1}^{6} \rho \iint_{S_B} \frac{\partial \phi_i}{\partial n}\phi_j ds \tag{3.80}$$

Substituting equations (3.67) and (3.68) into (3.66) gives the coefficient of restoring forces as;

$$F_{ij} = -\left(\frac{1}{\omega^2} \operatorname{Re} \sum_{i=1}^{6}\sum_{j=1}^{6} \rho \iint_{S_B} \frac{\partial \phi_i}{\partial n}\phi_j ds\right)\ddot{\eta} - \left(\frac{1}{\omega} \operatorname{Im} \sum_{i=1}^{6}\sum_{j=1}^{6} \rho \iint_{S_B} \frac{\partial \phi_i}{\partial n}\phi_j ds\right)\dot{\eta} \tag{3.81}$$

Where $\ddot{\eta}$ and $\dot{\eta}$ are the wave acceleration and velocities respectively. The restoring coefficient has a negative sign since the body is trying to bring back its equilibrium position.

In this study, a floating rectangular box was used in obtaining wave restoring forces. Its dimensions were taken to be, Length, L= 2.25m, Breadth, B=2.25m and the Draught, D to be 1.00m. These measurements were taken as those used by Ngina *et al.* (2015), who worked on wave exciting forces, for ease of validation. A Fortran code was then generated and the program included the mathematical formulations that had initially been derived. The results from this study are in agreement with Ngina *et al.* (2015) who analyzed wave exciting force and those of Endo (1987) who worked on the scattering effect on shallow waters.

CHAPTER FOUR

RESULTS AND DISCUSSION

The unending cases of washing away and capsizing of offshore floating structures have prompted more studies to be done on the same. That motivated this study to be done and the following are graphs for discussion.

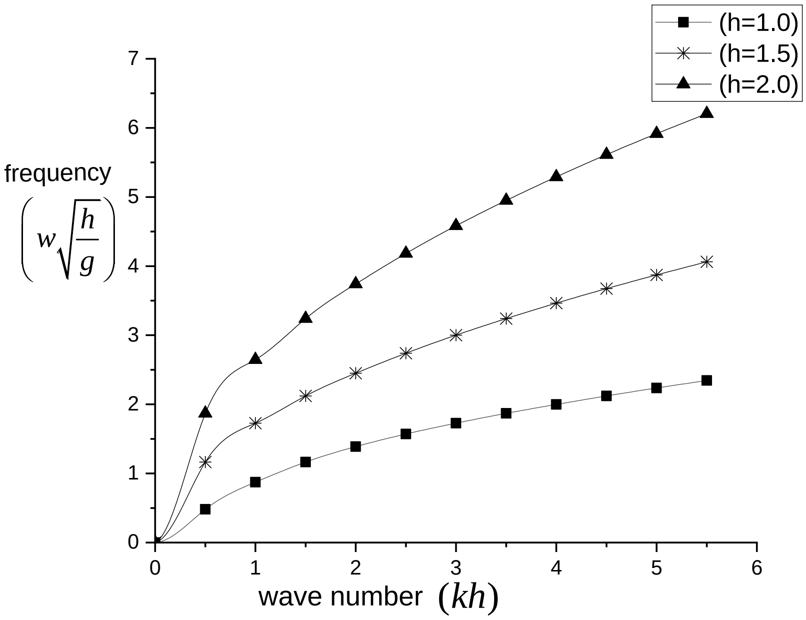

Figure 4.1: Dispersion relation of incident waves at different water depth

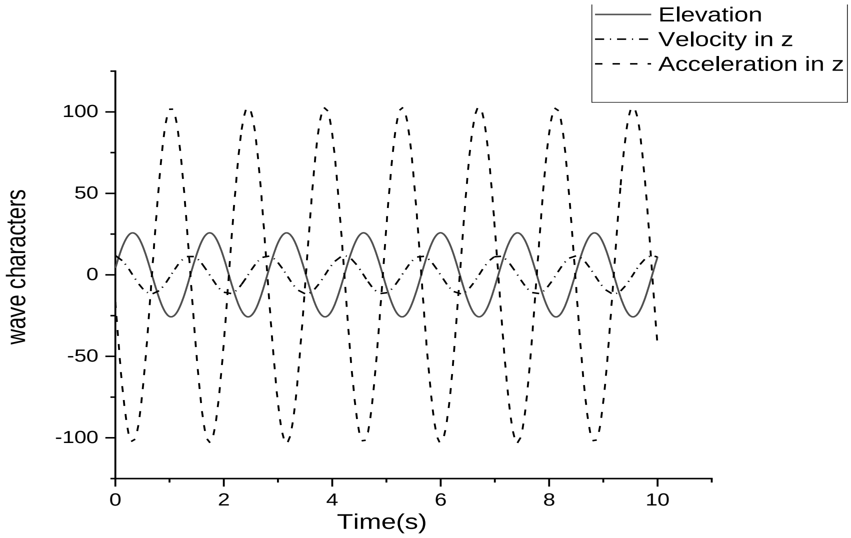

Figure 4.2: Relationship between wave elevation, velocity and acceleration in z-direction for water depth h = 2.0

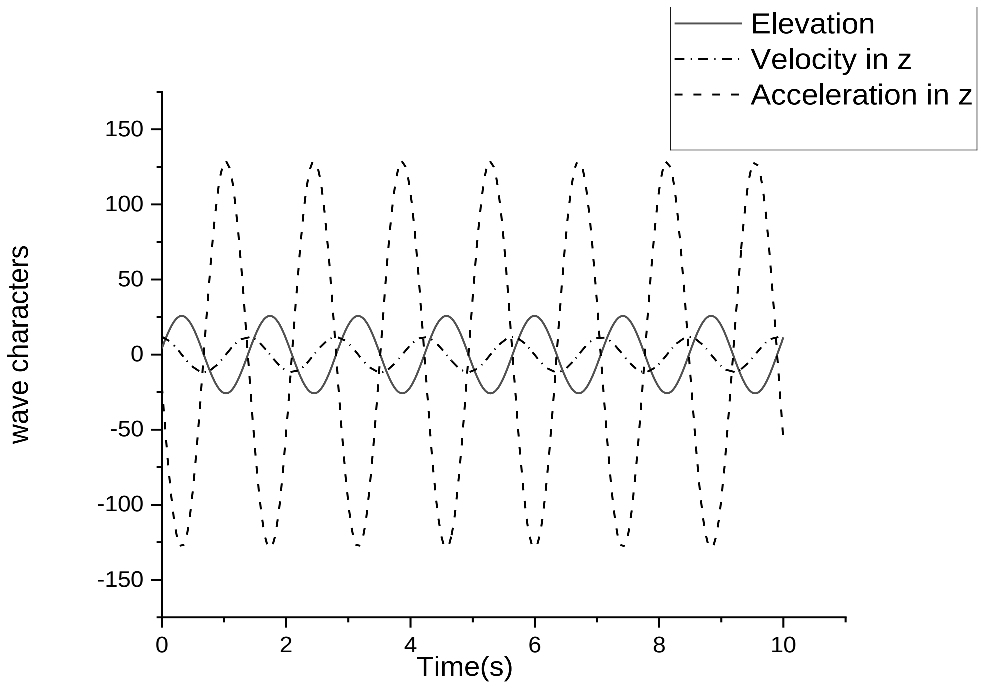

Figure 4.3: Relationship between wave elevation, velocity and acceleration in z-direction for water depth h = 2.5

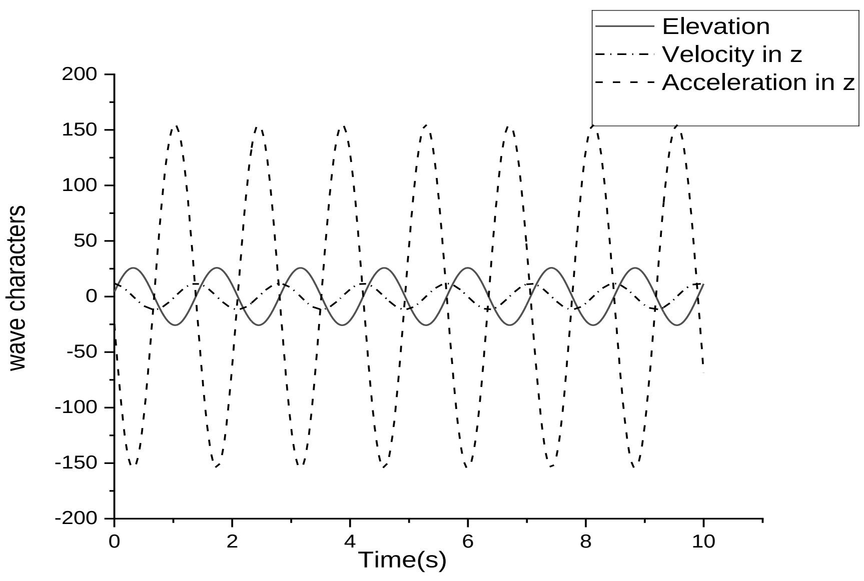

Figure 4.4: Relationship between wave elevation, velocity and acceleration in z-direction for water depth h = 3.0

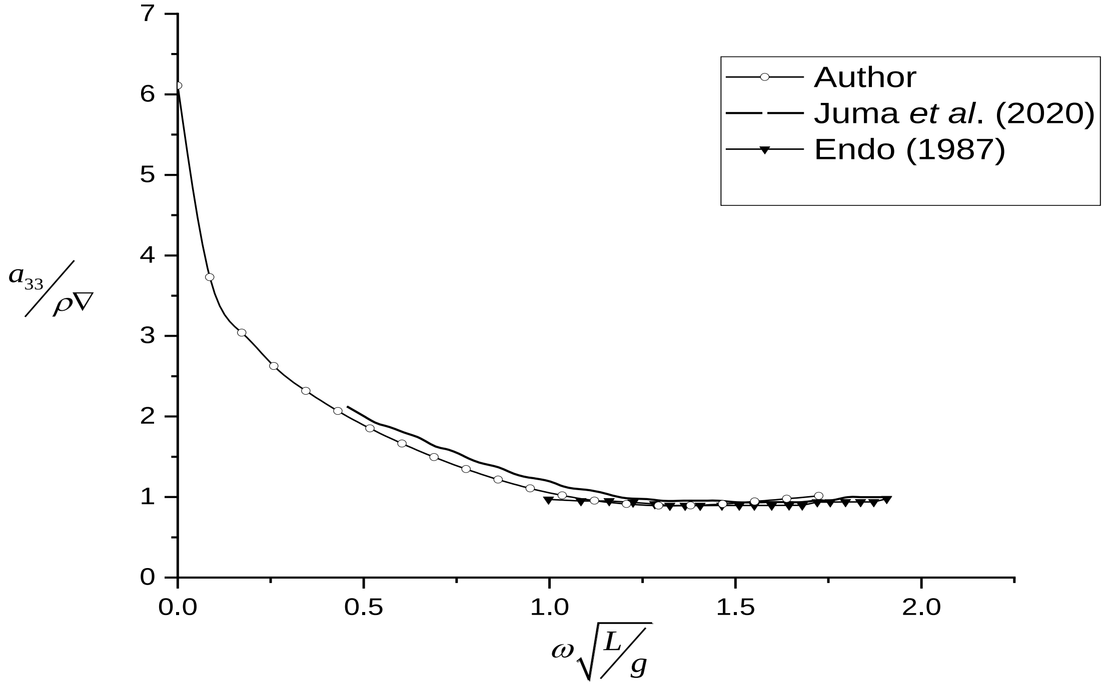

Figure 4.5: Non-dimensional added mass for heave against non-dimensional frequency

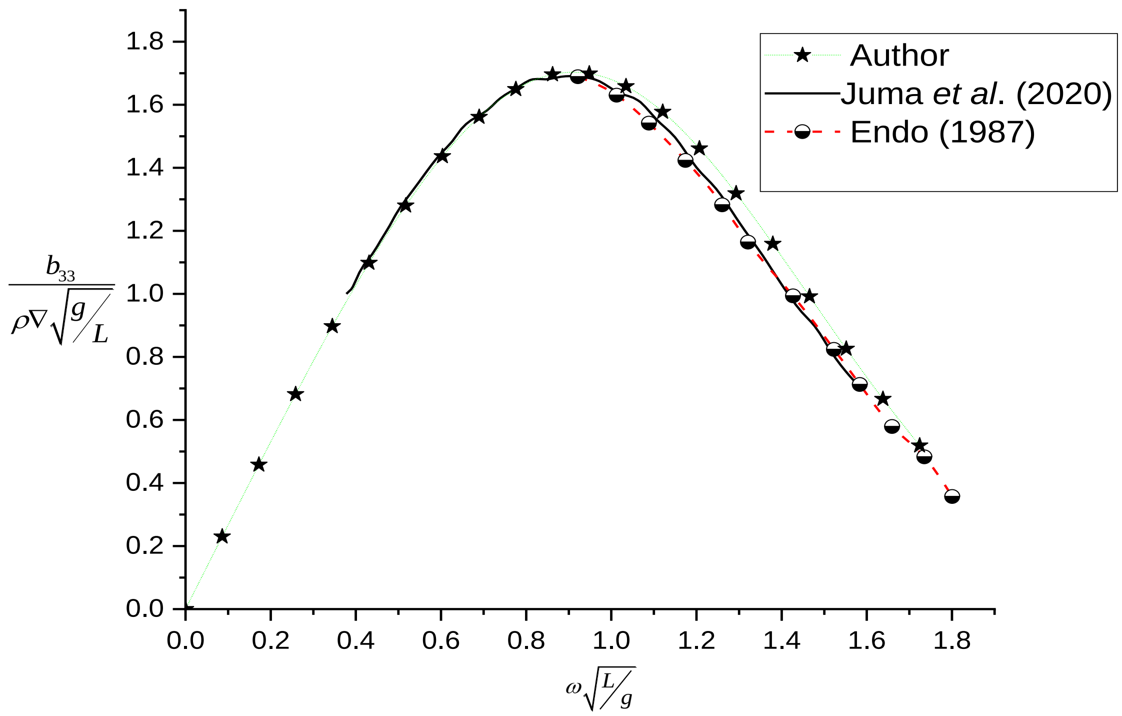

Figure 4.6: Non-dimensional damping for heave against non-dimensional frequency

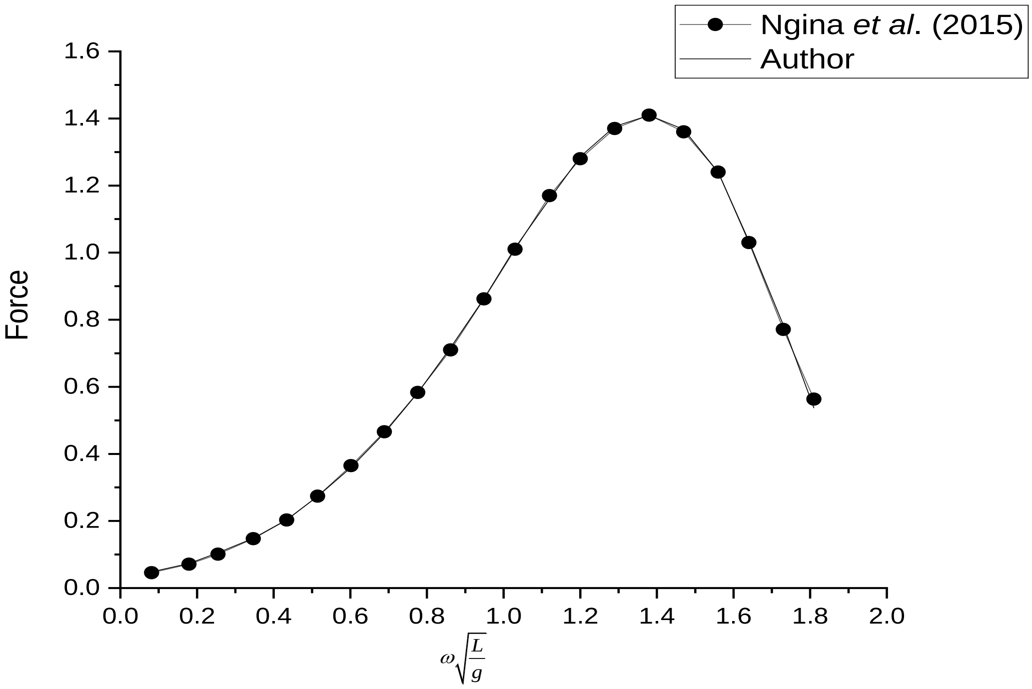

Figure 4.7: Restoring force for surge against non-dimensional frequency

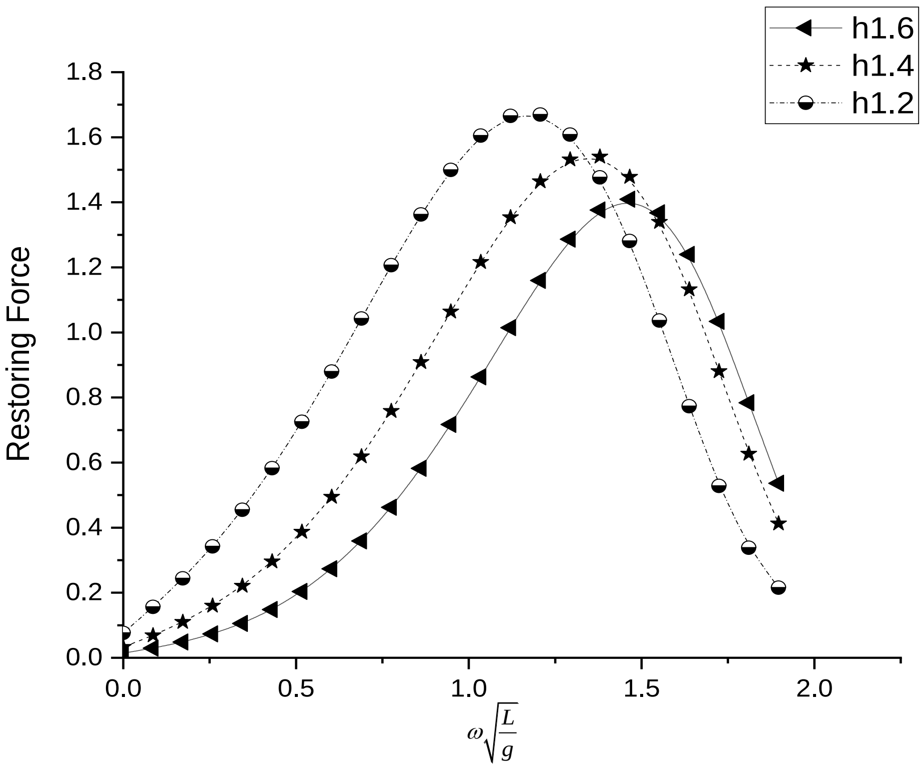

Figure 4.8: Non-dimensional restoring force for surge against non-dimensional frequency at varying water depth

As a way of getting proper understanding of wave restoring forces, it is important to have a clear understanding of dispersion relation and how wave characters change with respect to change in water depth. Since wave characters of incoming and outgoing waves are scientifically taken to be same, the characteristics of incident waves were used in this research to predict the characteristics of outgoing waves. Figure 4.1 is a representation of dispersion relation of the wave frequencies and wave numbers under varying water depths. Analysis of dispersion relation is of great importance in this study since the motion under study is oscillatory and is quite easier to analyze it using the frequency domain approach as compared to the time domain approach which will need step functions. Dispersion relation brings the relationship between wave frequency and the wavelength of the wave which is of importance to offshore engineers. It is evident from the graph that waves around smaller values of h are less dispersive. Otherwise, an increase in values of h increases wave dispersion. This is in agreement with theories of shallow waters where shallow water waves are less dispersive because water depths at these levels are shorter than the wavelengths. This therefore enables the waves to travel at the same speed.

The results explain why waves away from the shores always rise at greater heights as compared to the waves near the shores that travels in groups. These results are therefore of importance to offshore engineers in ensuring that floating structures are not placed so far from the shores where wave dispersion is high. Otherwise, if need be, they should be modelled with high restorative forces to counter the effects of high wave dispersion.

Figures 4.2, 4.3, 4.4 are a representation of the relationships between wave elevation, velocity and acceleration with varying depths from the shores. It is evident, from figures 4.2, 4.3 and 4.4, that with changes in depth the wave accelerations keep changing while elevation and velocity remains constant. From these figures it is worth noting that velocity and displacement are 90° out of phase while displacement and acceleration are 180° out of phase and this is true from the theories of physics of water waves. It is also good to note that the amplitudes of wave acceleration are quite higher than those of wave velocity.

The changes in wave acceleration that is depicted in these graphs are because of the changes in water frequencies with distances away from the shore. For instance, as the distance away from the shore increases, the wave frequencies increase and so does the wave amplitudes and this results in high accelerations. An increase in acceleration causes a subsequent increase in dispersion of the water waves and can in critical conditions result to standing waves which if not countered with high restoring forces, in the structures, results in capsizing or washing away of the structures.

Since acceleration of water waves is proportional to the acceleration of the structure, it therefore means that structures operating away from the shore experiences high accelerations which should be controlled by incorporating the knowledge of restoring forces in the design of these structures. These ensures that regardless of the high accelerations that are experienced by these structures, they still can assume their equilibrium positions because of the presence of restorative forces.

Figure 4.5 is a representation of heave added mass against frequency for a heaving rectangular box. The x-axis is a representation of non-dimensional frequency while the y-axis is a representation of heave added mass. From the graph, it is evident that with an increase in frequency, the heave added mass decays drastically. However, this is only evident up to some point that the decrease stops and the rate of heave added mass becomes constant. The results obtained are in agreement with those obtained by Juma *et al.* (2020) and that obtained by Endo (1987).

Figure 4.6 is a representation of heave damping coefficients against non- dimensional frequency. The x-axis is a representation of non-dimensional frequency while the y-axis is a representation of heave damping for a heaving rectangular box. From the graph, it is evident that heave damping increases with an increase in frequency up to a point that it acquires a maximum turning point and then begins to drop drastically with a further increase in frequency. This is in agreement with the literature on Wehausen and Laitone (1960) that heave damping goes to zero as frequency approaches infinity. The results obtained are in full agreement with those obtained by Juma *et al.* (2020) and confirms with the model of Endo (1987).

From figure 4.7, it is observed that the surge restoring force keeps on increasing with an increase in frequency until it attains a maximum turning point where the restoring force decays. As a way of bringing comparison between the exciting and the restoring force, the data of this work was validated against those of Ngina *et al.* (2015) and the results are seen to converge and in full agreement. This therefore fully satisfies Newtons third law of motion. The action in this case being the wave exciting force and its reaction being the wave restoring force. In nature, forces occur in pairs and a force cannot be exerted on a body without the same body experiencing a force itself that tries to counter that force which is acting on it. This therefore brings the idea that for an offshore structure to maintain its equilibrium position, it should be ensured that the magnitude of restoring force should be equal to the magnitude of the exciting force that makes the structures to oscillate. The intent of this work was to show that in order to maintain the general stability of an offshore structure, it is important to ensure that the restoring forces of the structure are modelled in a way that ensures that they are going to equally counter the exiting forces acting on them and ensure general stability, workability and safety of the structure.

Figure 4.8 shows the relationship between water depth and the intensity of restoring force. This brings comparison on the intensities of restoring forces at different depths. It is observed from the graph that with an increase in depth, there is a subsequent decrease in restoring force. This means therefore that towards the shores there are high restoring forces to counter the high exciting forces that are felt here due to the wave breaking that normally occurs. This is in agreement with the work done by Endo (1987) on scattering effect of shallow waters which explains why Tsunami waves are always so catastrophic and causes a lot of destructions. Respectively, it means that offshore structures that are operating in deep waters, with low restoring forces, should be modelled with inclusion of knowledge of restoring forces. This is because off the shore and in greater heights below the free surface, the waves are much stronger and causes the structures to oscillate. With low restoring forces, off the shores, these structures will not be able to maintain their equilibrium position and operate normally. That is why it is important that offshore engineers should incorporate the knowledge of restoring forces in the modelling of such structures to ensure its general stability, workability and safety of all those who are onboard. This is because the actual ocean environment is realistically very chaotic and structures undergo random movements which should be controlled by incorporation of knowledge on restoring forces in their modelling and design.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The chaotic ocean environment has always caused damages both to offshore structures and deaths to people working in them. Researchers in the field of hydrodynamics have been conducting research on hydrodynamic loads but their work has not been sufficient enough to handle the problem. In addition, offshore structures are subjected to different types of loading due to the continuous and unpredictable changes that happens in the ocean. Therefore, it is of importance to put key emphasis on restoring forces just like any other hydrodynamic load.

This research has added to the body of knowledge on wave loads and especially on restoring forces with an aim of having the problems of washing away and capsizing solved. In this study the restoring forces of a rectangular barge were analyzed for an offshore structure and magnitude of restoring forces varied at different depths. To do that, the incident wave characters were used as a prediction of the outgoing wave characters. Here, the Laplace equation was used as the governing equation hand in hand with the assumptions which were set both on the body surface and the free surface. This aided in the analysis of outgoing wave characters which are of importance in the prediction of wave restoring forces

Added mass and damping coefficients were derived from the integration of the Bernoulli equation over the wetted body surface. Wave restoring forces were then derived from putting together coefficients of added mass and damping and those of velocity and acceleration. The series form of the Greens function that was used by Ngina *et al.* (2015) was used in this study.

This research has clearly been able to bring out the relationship between wave exciting forces and the restoring forces. From the graphs, and in great agreement with Newtons third law of motion, it is clear that there is great relationship between exciting and restoring forces.

With this work, offshore engineers can now be keen on the modeling of offshore structures. That is, they should realize that as one moves deep into the sea, restorative forces decrease which therefore implies that the offshore structures should be modelled with adequate restoring forces. This will then ensure that these structures maintain their equilibrium positions amidst the strong effects of the waves.

5.2 Recommendations

Knowledge from this study can be used variedly. It can be used by offshore engineers in construction of structures that are dependable and highly resilient. The results obtained from this research can be used by offshore engineers who work on oil and drilling platforms in order for them to construct structures which have high restorative forces and which are efficient despite the rough nature of the sea environment.

The research can be extended further to the analysis of wave restoring moments for a rectangular barge oscillating at zero forward speed.

REFERENCES

Ahmed, M. O., Yenduri, A., & Kurian, V. J. (2016). Evaluation of the dynamic responses of truss spar platforms for various mooring configurations with damaged lines. *Ocean Engineering*, *123*, 411-421. https://doi.org/10.1016/j.oceaneng.2016.07.004

Abyn, H., Maimun, A., Jaswar, J., Islam, M. R., Magee, A., Bodaghi, B., & Tofa, M. M. (2014). Hydrodynamic interaction of floating structure in regular waves. *Jurnal Teknologi*, *66*(2), 91-96. https://doi.org/10.11113/jt.v66.2491

Asim, T., Islam, S. Z., Hemmati, A., & Khalid, M. S. U. (2022). A review of recent advancements in offshore wind turbine technology. *Energies*, *15*(2), 1-30. https://doi.org/10.3390/en15020579

Awal, Z. I., Mehtaj, N., & Pranto, R. I. (2021, November). Heave motion prediction of rectangular floating barge using artificial neural network. In *ASME International Mechanical Engineering Congress and Exposition* (Vol. 85611, p. V07AT07A052). American Society of Mechanical Engineers. https://doi.org/10.1115/IMECE2021-73311

Bhattacharyya, B. K. (1978). Computer modeling in gravity and magnetic interpretation. *Geophysics*, *43*(5), 912-929. https://doi.org/10.1190/1.1440873

Boccia, V., Renga, A., Rufino, G., D'Errico, M., Moccia, A., Aragno, C., & Zoffoli, S. (2015). Linear dispersion relation and depth sensitivity to swell parameters: Application to synthetic aperture radar imaging and bathymetry. *The Scientific World Journal*, *2015*(1), 374579. https://doi.org/10.1155/2015/374579

Cardo, A., Francescutto, A., & Nabergoj, R. (1984). Sub harmonic oscillations in nonlinear rolling. *Ocean Engineering*, *11*(6),663-669. https://doi.org/10.1016/0029-8018(84)90007-6

Cardoso, J., Vieira, M., Henriques, E., & Reis, L. (2021). Computational analysis of the transportation phase of an innovative foundation for offshore wind turbine. *Ships and Offshore Structures*, *16*(7), 725-734. https://doi.org/10.1080/17445302.2020.1779016

Cruz, J. (2007). *Ocean wave energy: current status and future prespectives*. Springer Science & Business Media.

Deng, S., Xu, Y., Ren, H., Fu, S., Li, S., Moan, T., & Gao, Z. (2022). Numerical simulation of wave-induced hydroelastic response and flow-induced vibration of a twin-tube submerged floating tunnel. *Marine Structures*, *82*(1), 103-124. https://doi.org/10.1016/j.marstruc.2021.103124

Deo, D. M. (2013). *Waves and structures*. Mumbai: Indian Institute of Technology Bombay.

De Vos, A., Aluwihare, L., Youngs, S., DiBenedetto, M. H., Ward, C. P., Michel, A. P., ... & James, B. D. (2021). The M/V X-Press Pearl Nurdle Spill: Contamination of burnt plastic and unburnt nurdles along Sri Lanka's beaches. *ACS Environmental Au*, *2*(2), 128-135. https://doi.org/10.1021/acsenvironau.1c00031

Endo, H. (1987). Shallow-water effect on the motions of three-dimensional bodies in waves. *Journal of Ship Research, 31*(1), 34-40. https://doi.org/10.5957/jsr.1987.31.1.34

Falnes, J., & Kurniawan, A. (2020). *Ocean waves and oscillating systems: linear interactions including wave-energy extraction* (Vol. 8). Cambridge university press.

Faltinsen, O. (1974). Motions of Large Structures in Waves at Zero Froude Number. In *International Symposium on the Dynamics of Marine Vehicles and Structures in Waves*, *90*(1), 91-106.

Faltinsen, O. (1990). Wave loads on offshore structures. *Annual Review of Fluid Mechanics, 22*(1), 35-56. https://doi.org/10.1146/annurev.fl.22.010190.000343

Gnitko, V. I., Karaiev, A. O., Myronenko, M. L., & Strelnikova, E. A. (2021). BEM analysis of gravitational–capillarity waves on free surfaces of compound shells of revolution. *International Journal of Computational Methods and Experimental Measurements, 9*(1), 38-50. https://doi.org/10.2495/CMEM-V9-N1-38-50

Hess, J. L., & Smith, A. M. O. (1964). Calculation of nonlifting potential flow about arbitrary three-dimensional bodies. *Journal of Ship Research*, *8*(4), 22-44. https://doi.org/10.5957/jsr.1964.8.4.22

Juma, C. M., Manyanga, D. O., & Gathia, P. W (2020). Radiation of linear waves for a heaving rectangular box in shallow water. *Internnational Journal of Scientific & Engineering Research 11*(3), 1051-1053.

Khan, N. A., Sulaiman, M., Tavera Romero, C. A., Laouini, G., & Alshammari, F. S. (2022). Study of rolling motion of ships in random beam seas with nonlinear restoring moment and damping effects using neuroevolutionary technique. *Materials, 15*(2), 674-695. https://doi.org/10.3390/ma15020674

Kianejad, S., Enshaei, H., Duffy, J., & Ansarifard, N. (2018). Calculation of restoring moment in ship roll motion through numerical simulation. *Paper Presented at the Proceedings of the 13th International Conference on the Stability of Ships and Ocean Vehicles, 16*(21), 429-442.

Kim, J. S., & Nam, B. W. (2022). Numerical analysis for hydrodynamic performance of OWC devices with multiple chambers in waves. *Journal of Ocean Engineering and Technology*, *36*(1), 21-31. https://doi.org/10.26748/KSOE.2021.091

Lamb, H. (1932). *Hydrodynamics, 5th edition.*, Cambridge: Univ. Press.

Li, A. J., Fang, H., & Liu, Y. (2022). Hydroelastic analysis of interaction between water waves and a floating laminated disk. *Physics of Fluids, 34*(4), 1-19. https://doi.org/10.1063/5.0088675

Liu, Z. C., & Soares, C. G. (2022). Numerical study on the mooring force of a gravity-type fish cage under currents and waves. *Trends in Maritime Technology and Engineering Volume 2*(1), 591-597.

Massel, S. R. (1996). *Ocean surface waves: Their physics and prediction* (Vol. 11). World Scientific.

Mendes, P., Correia, J. A., De Jesus, A. M., Ávila, B., Carvalho, H., & Berto, F. (2021). A brief review of fatigue design criteria on offshore wind turbine support structures. *Frattura ed Integrità Strutturale*, *15*(55), 302-315. https://hdl.handle.net/11250/3044644

Moan, T. (1997). Current trends in the safety of offshore structures. In *The Proceedings of the International Offshore and Polar Engineering Conference, 4*, 1-12.

Newman, J. N. (1992). Panel methods in marine hydrodynamics. In *Proc. Conf. Eleventh Australasian Fluid Mechanics*, *11*(2), 123-129

Newman, J. N. (2018). *Marine hydrodynamics*: The MIT press

Ngina, P. M., Manyanga, O. D., & Kaguchwa, J. N. (2015). Wave exciting force on a floating rectangular barge due to surface waves. *International Journal of Scientific & Engineering Research*, *6*(6), 1480-1485. http://41.89.96.81:8080/xmlui/handle/123456789/1427

Peşman, E., & Taylan, M. (2012). Influence of varying restoring moment curve on parametric roll motion of ships in regular longitudinal waves. *Journal of Marine Science and Technology*, *17*, 511-522.

Rouhan, A. & Schoefs, F. (2003). Probabilistic modeling of inspection results for offshore structures. *Structural Safety, 25*(4), 379-399. https://doi.org/10.1016/S0167-4730(03)00016-X

Shukla, A. & Karki, H. (2016). Application of robotics in offshore oil and gas industry—A review Part II. *Robotics and Autonomous Systems, 75*(2), 508-524. https://doi.org/10.1016/j.robot.2015.09.013

Stepanyants, Y., & Sturova, I. (2021). Hydrodynamic forces exerting on an oscillating cylinder under translational motion in water covered by compressed ice. *Water*, *13*(6), 822-843. https://doi.org/10.3390/w13060822

Surendran, S., & Reddy, J. V. R. (2003). Numerical simulation of ship stability for dynamic environment. *Ocean Engineering*, *30*(10), 1305-1317. https://doi.org/10.1016/S0029-8018(02)00109-9

Taylan, M. E. T. İ. N. (1996). Nonlinear roll motion of ships in beam waves. *Bulteni-Istanbul Teknik Universitesi*, *49*, 459-480. http://dx.doi.org/10.1016/S0029-8018(98)00064-X

Tanaka, Y., & Hudspeth, R. T. (1988). Restoring forces on vertical circular cylinders forced by earthquakes. *Earthquake Engineering & Structural Dynamics*, *16*(1), 99-119. https://doi.org/10.1002/eqe.4290160108

Tang, K., Wang, J. X., Chen, X., Jiang, D. P., & Li, Y. L. (2021). Nonlinear ship motion with forward speed in waves based on 3D time domain hybrid Green function method. *Engineering Analysis with Boundary Elements*, *123*(1), 107-121. https://doi.org/10.1016/j.enganabound.2020.11.006

Toffoli, A., & Bitner-Gregersen, E. M. (2017). Types of ocean surface waves, wave classification. *Encyclopedia of Maritime and Offshore Engineering*, *1*(14), 1-8. https://doi.org/10.1002/9781118476406.emoe077

Tromans, P. S., & Vanderschuren, L (2017) . Wave loads on offshore structures. *7th International Conference on "Experiments/ Process/ Systems Modelling/ Simulations/ optimization.*

Truong, T. (2022). *Steady waves in local and nonlocal models for water waves* (Doctoral dissertation, Lund University).

Wehausen, J. V., & Laitone, E. V. (1960). Surface waves. In *Fluid Dynamics/Strömungs Mechanik* (pp. 446-778): Springer.

Yong-Can, C., Zheng-Wen, L., Zhao-Wei, L., Xiao, C., & Hao-Ran, W. (2022). An implicit method for slow-moving free-surface boundary of open channel flows. *Journal of Hydraulic Research*, *60*(4), 619-627. https://doi.org/10.1080/00221686.2021.2022030

Yu, L., Ma, N., & Wang, S. (2019). Parametric roll prediction of the KCS containership in head waves with emphasis on the roll damping and nonlinear restoring moment. *Ocean Engineering*, *188*, 106298. https://doi.org/10.1016/j.oceaneng.2019.106298

Zheng, X. (1988). *Prediction of Motion and Wave Load of Mono and Twin Hull Ships in Waves.* ProQuest Dissertations & Theses.

Zhou, Y., Ning, D., Liang, D., & Cai, S. (2021). Nonlinear hydrodynamic analysis of an offshore oscillating water column wave energy converter. *Renewable and Sustainable Energy Reviews*, *145*(1), 1-15. https://doi.org/10.1016/j.rser.2021.111086

APPENDICES

Appendix I: Publication

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ISSN 2229-5518

Analysis of Surface Wave Characters in the Prediction of Wave Restoring Forces

Kibet Hillary, David O. Manyanga, Patricia W. Gathia.

Abstract

Floating and offshore structures are prone to water waves and are affected by waves in different ways. These structures generate their own waves when oscillating in water. The generated waves have varied impacts on the structures and on other offshore bodies. The commonly known impact is the creation of periodic loads which are the added mass, damping and the restoring moments and forces. Restoring forces act on the body to bring it back to the steady equilibrium state. Restoring forces are therefore important to ocean and marine engineers in modelling of both offshore and floating bodies. To date, very little has been done on this area. In fact, nobody has come up with a connectivity between the outgoing wave characters and the restoring forces. This work is concerned about the outgoing wave characters in the production of corresponding restoring forces. To do this, it is scientifically correct to say that the incoming and outgoing waves have the same characters and due to this similarity, the wave characters of incoming waves have the same characters as those of outgoing. Such to be analyzed include; wave elevation, velocity and vertical acceleration. The aim of this paper, therefore, is to analyze these wave characters and determine how they behave at different heights away from the shore. It is of great importance to note that the analysis of these wave characters was a success because of the presence of a velocity potential which is derived by the method of separation of variables and by use of governing boundary conditions. This paper also touches on the dispersion relation of water waves and an analysis was done to show how these waves disperse at different depths on or away from the shore.

Appendix II: Research permit

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Date of Issue: 24/August/2023

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See overleaf for conditions

Appendix III: FORTRAN Code

```
program main
use surface_integral
use surface_coordinates
implicit none
!integer m,n,p

integer, parameter:: m=4
integer, parameter:: n=4
integer, parameter:: p=3
integer, parameter:: mm=2*(m-1)*(n-1)+2*(m-1)*(p-1)+2*(n-1)*(p-1)

real(8):: x(mm,3,4),x1(mm,3,4),xf1,yf1,zf1,xgs,ygs,zgs,xf2,yf2,zf2,Tx,Ty,Tz,St
real(8):: NN(mm,6),Pi,gr1,gr2
real(kind=8),external::Gh,Ght,Ghtt
complex(kind=8),external::Fh,Gx,Gy,Gz
integer:: i,j,k,t,Ht,Mt,r
real(8):: ADM(6,6),DAM(6,6),BSS,ASS(mm),ASS1(mm),ASS2(mm),RF(6,6)
real(8):: Aaa(mm,mm),Abb(mm,mm),RR1(mm,mm),RR2(mm,mm)
real(8):: Acc(mm,mm),Add(mm,mm),RR3(mm,mm),RR4(mm,mm)
real(8):: XN,XM,grav,qN,qK,h,Hp,Hq,Hr,Lx,Ly,Lz,Dx,v,u,kl,q
real(8):: KN,KD,KC,KM

complex(8):: AA(mm,mm),BB(mm,mm),XX(mm)
complex(8):: AA1(mm,mm),FF1(mm,mm),BB1(mm)
real(8)::
Ar(mm,mm),Ai(mm,mm),Br(mm),Bi(mm),XXr(mm),XXi(mm),XXc(mm),As(mm,mm),Bs(
mm),Cs(mm,mm),Ds(mm)
real(8)::panel(3,4,(2*n*p+2*m*p+2*m*n)) ! the panel grids on the whole box

integer l
DIMENSION  JS(mm)
DOUBLE PRECISION js
```

```
call cpu_time(Tx)

!KN=0.01437007
!KN=0.005  !Surge
KN=0.00000125  !Heave
!KN=0.0651437007  !Common for Surge, Sway and Heave (To be used)
!KN=0.01  !Common for Surge, Sway and Heave (To be used)
!KN=0.0150538
!KN=0.0085
!!KN=0.06250538

!KM=0.00001437007
Pi=3.1415927
grav=9.81
Hp=90.0; Hq=90.0; Hr=40.0
Lx=Hp/Hr; Ly=Hq/Hr; Lz=Hr/Hr

kl=Lx
Dx=Lz
h=1.6*Dx

!gr1=-0.5*(grav/Lx)**0.5
!gr2=2.0*(grav/Lx)**0.5
gr1=0.0*(grav/Dx)**0.5
gr2=5.0*(grav/Dx)**0.5

call rect_surface(m,n,p,Lx,Ly,Lz,panel)
open(30,file='panel.txt')

do j=1,mm
            read(30,*)
x(j,1,1),x(j,2,1),x(j,3,1),x(j,1,2),x(j,2,2),x(j,3,2),x(j,1,3),x(j,2,3),x(j,3,3),x(j,1,4),x(j,2,4),x(j,3,
4)
            x1(j,1,1)=x(j,1,1); x1(j,2,1)=x(j,2,1); x1(j,3,1)=x(j,3,1)
```

```
        x1(j,1,2)=x(j,1,2); x1(j,2,2)=x(j,2,2); x1(j,3,2)=x(j,3,2)
        x1(j,1,3)=x(j,1,3); x1(j,2,3)=x(j,2,3); x1(j,3,3)=x(j,3,3)
        x1(j,1,4)=x(j,1,4); x1(j,2,4)=x(j,2,4); x1(j,3,4)=x(j,3,4)
enddo
close(30)

Open(43,file='mm.txt')
!Open(44,file='Result for ADM(k,t).txt')
!Open(45,file='Result for DAM(k,t).txt')
Open(46,file='Result for RF(k,t).txt')
!open(300,file='v.txt')
!open(301,file='XN.txt')
open(304,file='Tz.txt')



!do qN=gr1,gr2,0.2
!do qK=0.0099,10.2,0.2
!do qK=0.004,5.0 ,0.2  !1
do qK=0.00001,4.6,0.2  !2
    qN=qK
  !q=qN*sqrt(kl/grav)   !1
  !q=0.9524*qN*sqrt(kl/grav)  !2
  q=0.9*qN*sqrt(kl/grav)

   call Newton_KN(qN,grav,h,KD)


           KN=KD
           KM=KD
           XN=KN/h/(2.0*Pi)
           XM=KM/h/(2.0*Pi)
    v=XN*tanh(KN*h)
    u=XM*tan(KM*h)

!write(300,*) v
```

```
  do i=1,mm
        xf1=0.25*(x(i,1,1)+x(i,1,2)+x(i,1,3)+x(i,1,4))
        yf1=0.25*(x(i,2,1)+x(i,2,2)+x(i,2,3)+x(i,2,4))
        zf1=0.25*(x(i,3,1)+x(i,3,2)+x(i,3,3)+x(i,3,4))
        do j=1,mm
              call
guass_surface(Gh,Ght,Ghtt,Fh,Gx,Gy,Gz,xf1,yf1,zf1,x1(j,:,:),KN,KM,XN,XM,v,u,h,BB(i,j),
AA(i,j),ASS(j),ASS1(j),ASS2(j))
              call Aea1(xf1,yf1,zf1,x1(j,:,:),h,Aaa(i,j),Abb(i,j))
              call Aea2(xf1,yf1,zf1,x1(j,:,:),h,Acc(i,j),Add(i,j))
              call Normal_Direction(X1(j,:,:),NN(j,1:3))
              xf2=0.25*(x1(j,1,1)+x1(j,1,2)+x1(j,1,3)+x1(j,1,4))
              yf2=0.25*(x1(j,2,1)+x1(j,2,2)+x1(j,2,3)+x1(j,2,4))
              zf2=0.25*(x1(j,3,1)+x1(j,3,2)+x1(j,3,3)+x1(j,3,4))
              NN(j,4)=(yf1-yf2)*NN(j,3)-(zf1-zf2)*NN(j,2)
              NN(j,5)=(zf1-zf2)*NN(j,1)-(xf1-xf2)*NN(j,3)
              NN(j,6)=(xf1-xf2)*NN(j,2)-(yf1-yf2)*NN(j,1)
              RR1(i,j)=Aaa(i,j)
              RR2(i,j)=Abb(i,j)
              RR3(i,j)=Acc(i,j)
              RR4(i,j)=Add(i,j)
        enddo
  enddo
write(43,*) mm

        do i=1,mm
              do j=1,mm
                IF (i==j) THEN
                FF1(i,j)=-2.0*Pi
                ELSE
                FF1(i,j)=AA(i,j)
                END IF
         enddo
```

```
  enddo

  do k=1,3
        do i=1,mm
              BB1(i)=0.0
              do j=1,mm
                AA1(i,j)=FF1(i,j)+RR2(i,j)+RR4(i,j)
                BB1(i)=BB1(i)+(BB(i,j)+RR1(i,j)+RR3(i,j))*NN(j,k)
                !END IF
         enddo
  enddo

        do i=1,mm
              Br(i)=REAL(BB1(i)); Bi(i)=IMAG(BB1(i)); Bs(i)=0.0; Ds(i)=0.0
          do j=1,mm
              Ar(i,j)=REAL(AA1(i,j)); Ai(i,j)=IMAG(AA1(i,j));
As(i,j)=Ar(i,j)*Ar(i,j)+Ai(i,j)*Ai(i,j);
              Bs(i)=Bs(i)+Ar(i,j)*Br(i)+Ai(i,j)*Bi(i); Cs(i,j)=As(i,j);
Ds(i)=Ds(i)+Ar(i,j)*Bi(i)-Ai(i,j)*Br(i)
          enddo
        enddo
        CALL AGAUS(As,Bs,mm,XXr,L,JS)
        CALL AGAUS(Cs,Ds,mm,XXi,L,JS)

        do t=1,3
          ADM(k,t)=0.0; DAM(k,t)=0.0 ; RF(k,t)=0.0
              do j=1,mm
                    ADM(k,t)=ADM(k,t)+XXr(j)*ASS(j)*NN(j,t)  !0.2*
                    DAM(k,t)=DAM(k,t)+qK*XXi(j)*ASS(j)*NN(j,t)  !0.1*
                RF(k,t)=-RF(k,t)+ADM(k,t)*ASS2(j)-DAM(k,t)*ASS1(j)
              enddo
              !write(*,*) q,ADM(k,t),DAM(k,t)
              write(*,*) q,RF(k,t)
        enddo
```

```
enddo
                    !write(44,8) q,ABS(ADM(1,1)),ABS(ADM(2,2)),ABS(ADM(3,3))
                    !write(45,8) q,ABS(DAM(1,1)),ABS(DAM(2,2)),ABS(DAM(3,3))
                    write(46,8)
q,ABS(RF(1,1)/625)/1000*5.39125*2.0*0.5,ABS(RF(3,3)/625)/1000*5.39125
!write(301,*) q,KN
enddo

8       FORMAT(12F10.5,I3)

call cpu_time(Ty)
Tz=Ty-Tx
IF (Tz>=3600) THEN
Ht=Int(Tz/3600)
Mt=Int((Tz-3600*Ht)/60)
St=Tz-3600*Ht-60*Mt
ELSE IF (60<=Tz<3600) THEN
Ht=0
Mt=Int(Tz/60)
St=Tz-60*Mt
ELSE
Ht=0
Mt=0
St=Tz
END IF
write(*,*) 'the running time is', Ht, 'hours', Mt, 'mins', St, 'secs'
write(304,*)  Ht, 'hours', Mt, 'mins', St, 'secs'
!write(304,*) Tz
end program

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

function Gh(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)
implicit none
```

```
real(kind=8) Gh,xf1,yf1,zf1,xgs,ygs,zgs
real(8):: KN,KM,A,B,z,g,Pi,e,x,y,k,t
real(8):: XN,XM,v,h, u
!Gh=1.0
y=2.0
g=10
Pi=3.142
B=-Pi
e=0.1
k=KN !2
h=h
x=0.5
z=0.5
t=0.7125
!open(1,file='Gh.text')
!open(2,file='U.text')
!open(3,file='Ut.text')
A=sqrt((g*k)*tanh(k*h))
!do t=0,50.0,1.5
!U=-A*cos(B)*(cosh(k*h+k*y)/(5*sinh(k*h)))*cos(k*x*cos(B)+k*z*sin(B)-A*t)
!Ut=A*A*cos(B)*(cosh(k*h+k*y)/(50*sinh(k*h)))*sin(k*x*cos(B)+k*z*sin(B)-A*t)
                              !Along z-Axis
Gh=A*A*(cosh(k*h+k*z)/(k*10*sinh(k*h)))*cos(k*x*cos(B)+k*y*sin(B)-A*t)
!write(*,*)t,Gh
!write(1,*)t,n,U,Ut
!end do
!end program
end function

function Ght(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)
implicit none
real(kind=8) Ght,xf1,yf1,zf1,xgs,ygs,zgs
real(8):: KN,KM,A,B,z,g,Pi,e,x,y,k,t
real(8):: XN,XM,v,h, u
```

```
!Ght=1.0
y=2.0
g=10
Pi=3.142
B=-Pi
e=0.1
k=KN !2
h=h
x=0.5
z=0.5
t=0.7125
!open(1,file='Ght.text')
!open(2,file='U.text')
!open(3,file='Ut.text')
A=sqrt((g*k)*tanh(k*h))
!do t=0,50.0,1.5
!U=-A*cos(B)*(cosh(k*h+k*y)/(5*sinh(k*h)))*cos(k*x*cos(B)+k*z*sin(B)-A*t)
!Ut=A*A*cos(B)*(cosh(k*h+k*y)/(50*sinh(k*h)))*sin(k*x*cos(B)+k*z*sin(B)-A*t)
                              !Along z-Axis
Ght=A*A*A*(cosh(k*h+k*z)/(k*10*sinh(k*h)))*sin(k*x*cos(B)+k*y*sin(B)-A*t)
!write(*,*)t,Ght
!write(1,*)t,n,U,Ut
!end do
!end program
end function

function Ghtt(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)
implicit none
real(kind=8) Ghtt,xf1,yf1,zf1,xgs,ygs,zgs
real(8):: KN,KM,A,B,z,g,Pi,e,x,y,k,t
real(8):: XN,XM,v,h, u
!Ghtt=1.0
y=2.0
g=10
```

```
Pi=3.142
B=-Pi
e=0.1
k=KN !2
h=h
x=0.5
z=0.5
t=0.7125
!open(1,file='Ghtt.text')
!open(2,file='U.text')
!open(3,file='Ut.text')
A=sqrt((g*k)*tanh(k*h))
!do t=0,50.0,1.5
!U=-A*cos(B)*(cosh(k*h+k*y)/(5*sinh(k*h)))*cos(k*x*cos(B)+k*z*sin(B)-A*t)
!Ut=A*A*cos(B)*(cosh(k*h+k*y)/(50*sinh(k*h)))*sin(k*x*cos(B)+k*z*sin(B)-A*t)
                              !Along z-Axis
Ghtt=-A*A*A*A*(cosh(k*h+k*z)/(k*10*sinh(k*h)))*cos(k*x*cos(B)+k*y*sin(B)-A*t)
!write(*,*)t,Ghtt
!write(1,*)t,n,U,Ut
!end do
!end program
end function


function Fh(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)
!use IMSL
implicit none
real(8)::xf1,yf1,zf1,xgs,ygs,zgs
real(8)::y,Zxy,Vc,G1
real(8)::B,C,D,C1,C2,C3,CC,DD
real(8)::k5,k6,R1,R2,R3,R4,Pi
real(8):: KN,KM
real(8):: XN,XM,v,u,h,U1,U2
complex(kind=8)::Fh,A,AA
```

```
complex, parameter::  i=(0,1)
INTEGER, PARAMETER:: N=0
DOUBLE PRECISION X1,X3,J0,Y0,P0,Q0,MBSL1,MBSL2

Pi=3.1415927
k5=KN
k6=KM

        Zxy=(xf1-xgs)**2+(yf1-ygs)**2
        Vc=sqrt(Zxy)
        y=(Vc)**2
        R1=Zxy+(zf1-zgs)**2; R2=Zxy+(zf1+2.0*h+zgs)**2
        R3=sqrt(R1); R4=sqrt(R2)
        X1=k5*Vc
        X3=k6*Vc
   J0=MBSL1(N,X1)
        Y0=MBSL2(N,X1)
   P0=MBSL1(N,X3)
        Q0=MBSL2(N,X3)
        !U1=(zgs+h)/h; U2=(zf1+h)/h
        U1=(zgs+h); U2=(zf1+h)

              A=cosh(k5*U1)*(Y0+i*J0);B=cosh(k5*U2); C=2.0*Pi*(v*v-XN*XN);
D=(XN*XN)-(v*v)+v  !with h
              AA=-0.5*Pi*cos(KM*U1)*cos(KM*U2)*(Q0+i*P0);
CC=4.0*(XM*XM+u*u); DD=(h*XM*XM)+(h*u*u)-u
              C1=1.0/R3; C2=1.0/R4; C3=C2
              G1=C/D
  Fh=G1*B*A

end function

function Gx(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)  !x11=xf1, y11=yf1, z11=zf1
!use IMSL
```

```
implicit none
real(kind=8) xf1,yf1,zf1,xgs,ygs,zgs  !x11=xf1, y11=yf1, z11=zf1
real(8)::y,Zxy,Vc,Vd,G2
real(8)::B,C,D,C1,C2,C3,C31,CC,DD
real(8)::k5,k6,R1,R2,R3,R4,Pi
real(8):: KN,KM
real(8):: XN,XM,v,u,h,U1,U2
complex(kind=8)::Ax,Gx,AA
complex, parameter::  i=(0,1)
INTEGER, PARAMETER:: N=0
DOUBLE PRECISION
X1,X2,X3,X4,J0,Y0,P0,Q0,Mx,Nx,Ux,Vx,MBSL1,MBSL2,MBSL6,MBSL7

Pi=3.1415927
k5=KN
k6=KM

        Zxy=(xf1-xgs)**2+(yf1-ygs)**2
        Vc=sqrt(Zxy)
        Vd=(xf1-xgs)
        y=(Vc)**2
        R1=Zxy+(zf1-zgs)**2; R2=Zxy+(zf1+2.0*h+zgs)**2
        R3=sqrt(R1); R4=sqrt(R2)

        X1=k5*Vc
        X2=k5*k5*Vd
        X3=k6*Vc
        X4=k6*k6*Vd
   J0=MBSL1(N,X1)
        Y0=MBSL2(N,X1)
   P0=MBSL1(N,X3)
        Q0=MBSL2(N,X3)
        Mx=MBSL6(N,X1,X2)
        Nx=MBSL7(N,X1,X2)
```

```
            Ux=MBSL6(N,X3,X4)
            Vx=MBSL7(N,X3,X4)
            !U1=(zgs+h)/h; U2=(zf1+h)/h
            U1=(zgs+h); U2=(zf1+h)

                    Ax=cosh(k5*U1)*(Nx+i*Mx);B=cosh(k5*U2); C=2.0*Pi*(v*v-XN*XN);
D=(XN*XN)-(v*v)+v  !with h
                    AA=-0.5*Pi*cos(KM*U1)*cos(KM*U2)*(Vx+i*Ux);
CC=4.0*(XM*XM+u*u); DD=(h*XM*XM)+(h*u*u)-u
                    C1=1.0/R3; C2=1.0/R4; C3=C1+C2; C31=(-(xf1-xgs)/(R2*R4))
                    G2=C/D
                    Gx=G2*B*Ax

end function

function Gy(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)  !x11=xf1, y11=yf1, z11=zf1
!use IMSL
implicit none
real(kind=8) xf1,yf1,zf1,xgs,ygs,zgs  !x11=xf1, y11=yf1, z11=zf1
real(8)::y,Zxy,Vc,Ve,G2
real(8)::B,C,D,C1,C2,C3,C31,CC,DD
real(8)::k5,k6,R1,R2,R3,R4,Pi
real(8):: KN,KM
real(8):: XN,XM,v,u,h,U1,U2
complex(kind=8)::Ay,Gy,AA
complex, parameter::  i=(0,1)
INTEGER, PARAMETER:: N=0
DOUBLE PRECISION
X1,X2,X3,X4,J0,Y0,P0,Q0,My,Ny,Uy,Vy,MBSL1,MBSL2,MBSL6,MBSL7

Pi=3.1415927
k5=KN
k6=KM
```

```
        Zxy=(xf1-xgs)**2+(yf1-ygs)**2
        Vc=sqrt(Zxy)
        Ve=(yf1-ygs)
        y=(Vc)**2
        R1=Zxy+(zf1-zgs)**2; R2=Zxy+(zf1+2.0*h+zgs)**2
        R3=sqrt(R1); R4=sqrt(R2)

        X1=k5*Vc
        X2=k5*k5*Ve
        X3=k6*Vc
        X4=k6*k6*Ve
    J0=MBSL1(N,X1)
        Y0=MBSL2(N,X1)
    P0=MBSL1(N,X3)
        Q0=MBSL2(N,X3)
        My=MBSL6(N,X1,X2)
        Ny=MBSL7(N,X1,X2)
        Uy=MBSL6(N,X3,X4)
        Vy=MBSL7(N,X3,X4)
        !U1=(zgs+h)/h; U2=(zf1+h)/h
        U1=(zgs+h); U2=(zf1+h)

              Ay=cosh(k5*U1)*(Ny+i*My);B=cosh(k5*U2); C=2.0*Pi*(v*v-XN*XN);
D=(XN*XN)-(v*v)+v  !with h
              AA=-0.5*Pi*cos(KM*U1)*cos(KM*U2)*(Vy+i*Uy);
CC=4.0*(XM*XM+u*u); DD=(h*XM*XM)+(h*u*u)-u
              C1=1.0/R3; C2=1.0/R4; C3=C1+C2; C31=(-(yf1-ygs)/(R2*R4))
              G2=C/D
              Gy=G2*B*Ay

end function

function Gz(xf1,yf1,zf1,xgs,ygs,zgs,KN,KM,XN,XM,v,u,h)  !x11=xf1, y11=yf1, z11=zf1
!use IMSL
```

```
implicit none
real(kind=8) xf1,yf1,zf1,xgs,ygs,zgs  !x11=xf1, y11=yf1, z11=zf1
real(8)::y,Zxy,Vc,G2
real(8)::B,C,D,C1,C2,C3,C31,CC,DD
real(8)::k5,k6,R1,R2,R3,R4,Pi
real(8):: KN,KM
real(8):: XN,XM,v,u,h,U1,U2
complex(kind=8)::A,Gz,AA
complex, parameter::  i=(0,1)
INTEGER, PARAMETER:: N=0
DOUBLE PRECISION X1,X3,J0,Y0,P0,Q0,MBSL1,MBSL2

Pi=3.1415927
k5=KN
k6=KM

        Zxy=(xf1-xgs)**2+(yf1-ygs)**2
        Vc=sqrt(Zxy)
        y=(Vc)**2
        R1=Zxy+(zf1-zgs)**2; R2=Zxy+(zf1+2.0*h+zgs)**2
        R3=sqrt(R1); R4=sqrt(R2)

        X1=k5*Vc
        X3=k6*Vc
   J0=MBSL1(N,X1)
        Y0=MBSL2(N,X1)
   P0=MBSL1(N,X3)
        Q0=MBSL2(N,X3)
        !U1=(zgs+h)/h; U2=(zf1+h)/h
        U1=(zgs+h); U2=(zf1+h)

              A=cosh(k5*U1)*(Y0+i*J0); B=k5*sinh(k5*U2); C=2.0*Pi*(v*v-XN*XN);
D=(XN*XN)-(v*v)+v  !with h
```

```
                    AA=-0.5*Pi*KM*cos(KM*U1)*sinh(KM*U2)*(Q0+i*P0);
CC=4.0*(XM*XM+u*u); DD=(h*XM*XM)+(h*u*u)-u
                    C1=1.0/R3; C2=1.0/R4; C3=C1+C2; C31=(-(zf1+2.0*h+zgs)/(R2*R4))
        G2=C/D
        Gz=G2*B*A

end function



!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
SUBROUTINE Newton_KN(qN,grav,h,KD)
 implicit none

INTEGER N
REAL(8):: KN,KD,y
real(8):: x,qN,grav,h,CN,DN,EN,FN
Open(450,file='Result for DTO.dat')

  x=(qN*qN/grav)*h
  IF (x<=2.0) THEN
  KN=sqrt(x)*(0.994+0.1701*x+0.0305*x*x)
  ELSE
  KN=x+2.0*x*exp(-2.0*x)-6.0*x*x*exp(-4.0*x)
  END IF

  y=KN
  CN=tanh(KN*h); DN=cosh(KN*h); EN=sinh(KN*h); FN=sinh(2.0*KN*h)

N=1
DO WHILE (ABS( F(y) ) >= 1E-6 .AND. N < 5000)

  y=y-(F(y)/DF(y))

 N=N+1
```

```
 !RETURN
END DO
KD=y/h
 CONTAINS

  FUNCTION F(y)
   REAL(8):: y, F
   F=0.5*LOG(10.0*(y+x)/(y-x))-y
  End FUNCTION F

  FUNCTION DF(y)
   REAL(8):: y, DF
   DF=-(x/(y*y-x*x))-1.0
write(450,*) qN,KN
  End FUNCTION DF

 End SUBROUTINE Newton_KN
```