ANALYSIS OF WAVE EXCITING FORCES ON A FLOATING RECTANGULAR BARGE AT ZERO FORWARD SPEED

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DECLARATION AND RECOMMENDATION

DECLARATION

This thesis is my original work and has not been submitted or presented in part or whole for examination in any institution.

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RECOMMENDATION

This thesis has been submitted with our approval as supervisors for examination according to Egerton University regulations.

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DEDICATION

I dedicate this dissertation to my mother Lydiah Ngina and my brother Boniface Ngina. Their love, encouragement, social and academic support motivated me, and shaped me into the person I am.

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ABSTRACT

Surface waves have significant effects on the hydrodynamics of offshore bodies or structures on a fluid of finite depth. Wind, moving vessels, seismic disturbances of shallow sea floors (tsunamis) and the gravitational disturbances of the sun and the moon are factors responsible for generation of waves. Their influence is very crucial in engineering analysis, design, and optimization. Many researchers in the field of hydrodynamics have analyzed the effect that surface waves have on bodies with cylindrical cross section. However, little has been done on structures that have rectangular cross-section especially on incompressible fluids. This study, therefore, focused on the analysis of wave motions acting on rectangular offshore structures. The analysis of surface waves characteristics arising from incident wave potential was carried out. This is because the ability to predict offshore structure behavior begins with the study of the nature of ocean in which the vessel operates. These characteristics included: incident wave potential, incident wave elevation, wave velocity and acceleration. The influence of water depth on the wave characteristics was also investigated. Consequently, the wave exciting force resulting from the interaction of such waves on a rectangular floating barge was investigated and analyzed. Boundary integral method together with Green functions in its series form was used to obtain the radiation potentials. These radiation potentials were used to solve diffraction and the Froude-Krylov forces for the rectangular floating barge. The forces aforementioned were used in the calculation and the analysis of wave exciting forces. Research finding shows that, the vertical wave acceleration and wave velocity were quite high leading to high vertically induced motions. Acceleration are used in the determination of cargo loads and also in the predictions of sea sickness. Change in water depth was also found to have adverse effect on the wave properties and consequently on the hydrodynamic forces, this was in accordance with shallow water effect. For the surge exciting forces it was evident that the forces increased up to a certain level and then they were radiated away to avoid interference at far field. Moreover heave wave exciting forces was found to be inversely proportional to the wave frequency. Heave motion is the limiting factor in drilling of oil. To reduce the heave forces acting on a body it was observed that there was need to increase the distance of the body from the free surface, that is, the draught length. The results obtained would be of great importance to Kenya in the prospects of exploitation of oil on the southern coast and indeed also in the sand harvesting process that is taking place at the Kenyan coast for the construction of the standard gauge railway.

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LIST OF ABBREVIATIONS AND ACRONYMS

BEM	Boundary Element Method
FEM	Finite Element Method

LIST OF SYMBOLS

A	Wave amplitude
c	Wave celerity
δ	Dirac delta function
F_I	Froude Krylov force
F_D	Diffraction force
F_{ex}	Wave exciting force
g	Gravitational acceleration
h	Fluid depth
H	Draft
i, j, k	Unit vectors along the x -, y -, z - axes, respectively.
k	Wave number
$\vec{n}$	Vector normal
n_x, n_y, n_z	Vector normal on the $x, y, z,$ respectively
n_k	Normal vector in the six degrees of freedom
$\eta(x, z, t)$	Variation of the free surface; surface elevation
η_D	Diffraction wave elevation
p	Atmospheric Pressure
Re	Real part
$\vec{r}$	Position vector of the surface dS in the (x, y, z) coordinates system
t	Time
u	Wave velocity in the x - direction
u_t	Wave acceleration in the x - direction
v	Wave velocity in the y -direction
v_t	Wave acceleration in the y -direction
V	Velocity field
w	Wave velocity in the z -direction

w_t	Wave acceleration in the z -direction
ω	Wave angular frequency
ρ	Fluid Density
ξ_j	Complex amplitude of the body motion
ϕ	Velocity potential
ϕ_A	Incident velocity potential and diffracted wave potential
ϕ_D	Diffracted velocity potential
ϕ_I	Incident velocity potential
ϕ_j	Response of a rigid body in absence of incident wave's that is; fluid disturbances due to the motion of the body, that is, the radiation potentials
$f_j(\xi,\eta,\zeta)$	Source strength function
J_O	Bessels functions of order zero
$G(x,y,z,\xi,\eta,\zeta)$	Green's function of a point wave source of unit strength located at a point
$P = p(\mathrm{x,y,z})$	A point on the surface of the body
$Q = q(\xi,\eta,\zeta)$	A source point
S	Boundaries of the entire fluid field
S_B	Body surface
S_∞	Body surface at infinity
θ	The angle between the incident wave and the axis's
(x,y,z)	Cartesian coordinates

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1.1 Background information

The prediction of sea keeping parameter, for example, body response, wave load, deck wetness, slamming, among others, are some of the most important aspects in ship design (Faltinsen, 2005; Manyanga, O.D., Duan, W.Y., Xuliang H., and Cheng, P., 2014). Furthermore, peak loads created by hurricanes winds and waves and fatigue loads generated by waves over the body are also important design consideration for offshore construction. Offshore platforms, which is part of offshore construction, have many uses including oil exploration and production, navigation, ship loading and unloading and to support bridges and causeways. However, offshore oil drilling is one of the most visible and imperative application. The structures aforementioned must function properly and safely for a longer period, although they are subject to harsh marine environment (Sadeghi, 2007). Therefore, sea keeping analysis is an imperative aspect that should be incorporated into the design phase of any offshore structure. If this analysis is considered in the design loop, it is, in principle, possible to construct efficient and safer offshore structures. In the classical view of ocean hydrodynamics, fluid density is always assumed to be constant (Koo and Kim, 2010). This study adopted the same single-layer fluid approach that had been used in other studies of a similar nature (Beck and King, 1989; Rahman and Bahtta, 1993; Fonesca and Soures, 2002; Dai and Duan, 2008) where the main point of divergence was a rectangular floating barge that was considered and also the fact that the problems were solved in time domain study but in this research the problem was solved in frequency domain.

Various studies have established that surface waves cause periodic loads on all man-made structures in the sea regardless of whether these structures are rigid or floating, or whether they are deep in the ocean or on the surface. These periodic loads are caused by the interaction between water waves and floating bodies. The study of these periodic loads has received considerable attention from designers. This attention is attributable to the fact that accurate predictions for the hydrodynamic loads are crucial in designing of any offshore structure and construction taking place in shallow fluids (Wehausen and Laitone, 1960; Zakaria, 2009; Manyanga and Duan, 2011). The hydrodynamic interaction of a floating body and surface waves which cause periodic loads, can be decomposed into the radiation problem (where the body undergoes oscillatory and translatory motion), and the diffraction problem (where the body is fixed and restrained from oscillating). Much of the available body of literature focuses on radiation and diffraction problems from the perspective that most, if not all, offshore structures are cylindrically shaped (Garrett, 1971; Yeung, 1981; Bhatta and Rahman, 2003; Hassan and Bora, 2012; Finnegan, W., Meere, M. and Goggins, J., 2013). From the previous studies, it is evident that there is a deliberate effort by researchers to understand and analyze the impulsive hydrodynamic loads acting on cylindrical or spherical bodies. However, little effort has been put in analyzing hydrodynamics loads acting on rectangular floating bodies especially in incompressible fluids. However, Gou, Y., Chen, X., Teng, B. and Zheng, Y. (2012) advice that it is not realistic to presuppose that all structures are and will always have a cylindrical cross-section. This study assessed the reaction of a floating rectangular barge in the presence of surface waves, where body motions were attributed to incident waves and the scattering of these waves. The study derived the wave exciting force acting on a rectangular floating barge at zero forward speed using panel methods developed by Hess and Smith (1964). Although Manyanga *et al*. (2014) explored the said method in the analysis of the wave exciting forces for a two layer fluid, the method had not been studied in analyzing the exciting forces for a fluid of constant density.

1.2 Statement of the problem

Analysis of wave interaction with bodies is an important and active field in hydrodynamics. Interaction of ocean waves with floating bodies lead to forces and moments. These forces significantly affect offshore structures and bodies. For instance, they cause additional resistance, reduced sustained speed, and increased fuel consumption for ships; and even in oil exploration, they reduce the amount of resources drilled at a time. However, many past studies focused on hydrodynamic forces on cylindrical offshore bodies. In addition, previous studies have not addressed the impact of hydrodynamic forces on bodies having the profile of a rectangular shape, on water of finite depth. Due to maritime technological advancement, many rectangular bodies are increasingly being used on the ocean. Therefore, this study analyzed the surface wave characteristics and consequently explored the impact of wave exciting forces on a rectangular floating barge at zero forward speed for a fluid of finite depth.

1.3 Objectives

1.3.1 General objective

To analyze the wave exciting forces on a rectangular floating barge at zero forward speed for a fluid of constant density.

1.3.2 Specific objectives

i) To analyze the incident surface wave characteristics in three-dimension for a fluid of finite depth and their effects on a rectangular floating barge.
ii) To determine the Froude-Krylov force acting on a rectangular floating barge due to the presence of surface waves.
iii) To determine the diffracted force acting on a rectangular floating barge due to the presence of surface waves.
iv) To investigate and analyze the wave exciting force on a rectangular floating barge.

1.4 Assumption of the study

i) The fluid to be considered is homogeneous, incompressible, inviscid and without surface tension.
ii) The motion of the fluid is irrotational and harmonic with time dependence.
iii) The bottom of the channel or the seabed is stationary, impermeable and horizontal.

1.5 Justification

Knowledge of wave-induced loads and motion of ships and offshore structures is very important both in design and operational studies. Presently, there is an increase in the advancement of ocean technology. Consequently, high-tech marine vehicles for transportation of goods and passengers have been constructed. Furthermore, ocean engineers have also been of help in open sea fishing, recovery of deep sea minerals, and development of marine energy resources. All these areas require the study of hydrodynamics loads due to wave-induced motion. Mostly, researchers in this field have analyzed these hydrodynamic loads using circular floating bodies. However, the fundamental question is whether or not it is realistic to assume that most of the bodies on the sea have circular cross-section. This study was premised on the fact that not all floating bodies are circular in nature. Therefore, in this study a rectangular floating barge was used in the analysis of the wave exciting force in the surge and heave direction. This research came at a time when Pan continental oil and Gas Company discovered oil and completed the drilling of sunbird-1 well off the southern coast of Kenya, which is the first offshore oil column to be discovered in East Africa. The development of this new offshore energy fields in Kenya is expected to bring in new technical and economic challenges, particularly in respect of transferring the oil and gas products from the seabed to the sea surface. In addition the construction of standard gauge railway has led to sand harvesting on the Kenyan Coast which is another area of application of hydrodynamic forces.

The equipment to be used in the exploitation of these resources are subject to the principles advanced in this study; that is, the study of waves and body response to them. In addition, the results obtained would be of great help to researchers and oceanographers since it would help them in understanding the response of floating rectangular bodies in the ocean such as ships due to surface waves. For instance, the results findings on the relative vertical motion could be used to evaluate the possibility of slamming, damage due to slamming and volume of the water on the deck. The results of this study can be used in analyzing the heave motion which the drillers and designers of drilling equipment can rely on and help them to design structures with low heave motion to ensure that it is possible to extract more minerals at a time. Furthermore, in case of ships the information would be used to choose optimum ship routes based on relevant criteria like minimum fuel consumption or the shortest time of voyage.

CHAPTER TWO

LITERATURE REVIEW

2.1 Waves

When the surface of a body of water is disturbed in the vertical direction, the force of gravity always tries to return it to equilibrium. The returning of surface water to equilibrium has inertia, which causes oscillation. This oscillation is responsible for the propagation of the waves. Some disturbing forces, therefore, generate waves in water surface. A wave motion can be defined as any fluid motion in a gravitational field and with a free surface on an interface. There are different types of waves brought about by the changes in their periods. These include waves generated by winds, moving vessels, seismic disturbances of shallow sea floors (tsunamis) and the gravitational disturbance of the sun and the moon. From these causes, arise different types of waves, including internal waves and surface waves. Internal waves may occur due to density differences of a fluid, caused by variation in salinity and/or temperatures (Gou *et al.,* 2012; Manyanga and Duan, 2012; Manyanga *et al.*, 2014). Surface waves are those that occur at the surface of the ocean and the fluid involved is of constant depth.

The effects of internal waves on the surface and internal wave amplitude and the influence of internal waves on wave characteristics such as wavelength, frequency and period had been studied (Manyanga and Duan, 2011). The analysis and prediction was of great practical significance. The importance is attributable to the fact that ocean surface waves are known to cause periodic loads on all offshore structures, whether these structures are fixed, floating or sailing and whether on the surface or deeper in the sea (Zakaria, 2009). To understand these loads, a good comprehension of the physics of water waves was necessary. The analysis of incoming wave characteristics is especially of great importance to ocean engineers and designers in understanding the wave exciting forces. For instance, Baarholm and Faltinsen (2004), while studying the extreme wave impact on the deck, acknowledged that it was important to design an offshore structure in such a way that the lower deck was above the predicted wave level. For wave impact on floaters, vertical forces are critical. Furthermore, the study predicted that impact forces might influence the vertical platform significantly. Due to the presence of large volume structures, the body's free surface wave elevation and even kinematics are disturbed. In ship wave elevation, which is one of the characteristics that was analyzed, contributed significantly in wave-deck problem. Linear theory (Coastal engineering research center, 1977) has been used and is still in use in the prediction of sea environment. This theory aids in giving a good analysis of surface waves and the wave induced motions. Over the years, researchers used linear theory in predicting the kinematic and the dynamic characteristics of waves and also the effects they have on offshore structures, especially diffraction problems (Salvesen, N., Tuck, E. O. and Faltinsen, O., 1970; Vasquez, G.A., Fonseca, N. and Soures, G. C., 2011). According to this theory, water is assumed to be inviscid, irrotational and incompressible, and the wave slope is assumed to be asymptotically small (Faltinsen, 1990). This assumption helps in giving a linear description of surface gravity waves of any homogeneous fluid. Furthermore, it helps in relating the wave-induced motion and the load amplitudes, which have been found to be linearly proportional (Faltinsen, 2005). This theory has generated good results, which are in agreement with the practical results.

2.2 Dispersion relation

Dispersion relation relates the wave number k , the angular frequency ω , and the wave celerity c . For a linear dispersion relation, k is regarded as eigen- value (Rahmann and Bhatta, 1993).

$$\omega = \sqrt{gk \tanh kh} \,, \tag{1}$$

However, the wave celerity c , is given by;

$$c = \frac{\omega}{k} \tag{2}$$

then substituting equation (1) into equation (2) we have

$$c = \sqrt{\left(\frac{g}{k}\tanh hk\right)} \tag{3}$$

These equations show that the wave celerity does not depend on the height of the wave or on the amplitude. In addition they uniquely relate the wave frequency and wave number given the water depth. Consequently, for waves with different periods, the one with a longer period propagates at high celerity and moves ahead unlike the one with shorter period. It can therefore be concluded that the wave frequency increases with depth (Manyanga and Duan, 2012). This shows that waves generated by seismic disturbances (tsunamis) move ahead of others at very high celerity and their effects are catastrophic as compared to other surface gravity waves. Wave dispersion and how changes in the fluid depth affects the wave angular frequency and wave number on the surface waves was analyzed in this research.

2.3 Boundary conditions

Analyses of waves and the effects they have on offshore structures have been made possible by certain conditions. This possibility is due to the fact that the distinction between fluid motion occurs due to the boundaries imposed on the fluid domain (Linton and Mclver, 2001; Malik, S.A., Guang, P., and Yanan, L., 2013). In particular, in order to complete the mathematical problem of deriving the velocity potential, these boundary conditions are important in aiding the work of researchers interested in the study of hydrodynamics loads.

2.3.1 Free surface conditions

Surface waves are created by the presence of the free surface. Therefore, in all problems involving waves, free surface conditions always play a major role. Two free surface conditions are successful in describing most practical situations of sea environment, namely dynamic free surface and the kinematic free surface condition (Bahtta and Rahman, 1993). The dynamic free surface condition requires that the pressure of the fluid be equal to the pressure on the free surface, which is always taken to be the atmospheric pressure (Herman, 2011). On the other hand kinematic free surface condition requires that the fluid particle remain on the free surface, that is, the particle follows the free surface, and the free surface follows the particle as long as the motion is smooth and the wave does not break. The label kinematic is derived from the fact that it addresses the motion of the fluids and proposes that the normal component of the fluid velocity must be equal to the normal component velocity on the free surface itself. The linear dynamic condition comes from the Bernoulli principles (Faltinsen, 1990).

$$p + \rho \frac{\partial \phi}{\partial t} + \frac{\rho}{2}\left(\nabla \phi\right)^2 + \rho g y = c \tag{4}$$

The Bernoulli's principle is a non-linear equation, and hence linear analysis is used, where the boundary conditions on the free surface and the body boundary are linear.

2.3.2 Radiation condition

This condition states that surface waves and hence the velocity potential, disappear at any distance away from the body. Hence, $\nabla \phi = 0$ as $r \to \infty$ (Faltinsen, 1990).

The boundary conditions highlighted here are of paramount importance. These boundary conditions were applied in the derivation of the velocity potentials and, consequently, the mathematical formulation of the Froude-Krylov and the diffraction forces.

2.4 Body response in waves

Behaviors of offshore structures operating in the ocean environment are influenced by various kinds of external loads such as incident waves, thermal change, traffic loads, currents, and buoyancy. For any structure to be designed properly, the hydrodynamics loads must be predicted accurately. Furthermore, vertical acceleration and the relative vertical motion between the floating body and the waves are very important responses. For instance, they have always been associated with slamming (impulse loads with high pressure peaks that occur between the ship and the water) (Baarholm and Faltinsen, 2004). One of the main reason for studying fluid motion and waves on a floating or a rigid body is to determine the forces and moments acting on the body; that is, the hydrodynamic loads. These loads are present due to many factors, especially due to wave excitation (Rahman and Bhatta, 1993). In the analysis of wave exciting force, incident waves were the most influential and most important in the determination of hydrodynamic forces especially the diffraction and the Froude Krylov forces.

When an offshore body encounters surface waves with amplitude A and direction θ incident upon the body, moved with response to these waves in six degrees of freedom, translatory motions, that is the heave, surge and sway and oscillatory angular motions about the same axis, which were referred to as yaw, roll and pitch (Baghfalaki and Samir, 2013). The resulting motion acting on these offshore structures according to linear theory was very small. The corresponding velocity was sinusoidal and the corresponding frequency equal to that of the incident waves. According to linear potential theory, the potential of a floating body can be expressed as a sum of the potential due the undisturbed incoming waves, the potential due to diffraction of the undisturbed incoming waves on the fixed body and the radiation potential due to the six body motions as shown in equation (5) below (Sarantopoulos, 2004; Nguyen' and Yeung, 2011; Marvrakos and Konispoliatis, 2012; Manyanga *et al*., 2014).

$$\phi = \sum_{j=1}^{6} \phi_j + \phi_I + \phi_D \tag{5}$$

It is worth noting that each potential function must satisfy all the important and appropriate boundary conditions. In addition the velocity potential can be written using the superposition principle shown in equation (6) which is another form of equation (5)

$$\phi(x,y,z) = \mathrm{Re}\left\{\left(\sum_{j=1}^{6} \xi_j \phi_j(x,y,z) + A\phi_A(x,y,z)\right)e^{i\omega t}\right\} \tag{6}$$

From the assumption of small amplitude motion then the kinematic boundary condition on the immersed surface of the body is satisfied in its mean position, that is; (Manyanga *et al.*, 2014),

$$\frac{\partial \phi_j}{\partial n} = i\omega t = n_j,\ j = 1,2,\ldots,6 \quad , \quad \frac{\partial \phi_j}{\partial n} = i\omega\left(\omega \times r\right)_{j-3},\ j = 4,5,6 \tag{7}$$

and

$$\frac{\partial \phi_A}{\partial n} = 0 \tag{8}$$

The waves generated by a moving body are absent if the floating body is permanently fixed. Bernoulli's principle is used in the calculation of hydrodynamic loads by the integration of the pressure. These hydrodynamic loads problem in regular waves are categorized into two cases:

i) The forces and moments on the body when the body is restrained from oscillating. The hydrodynamics loads considered are the wave exciting loads, which comprise the Froude-Krylov and diffraction forces and moments (Faltinsen, 1990).
ii) The forces and moments on the body when the structure is forced to oscillate with the wave excitation frequency in any rigid body mode. In this case, incident wave is not considered. The hydrodynamics loads comprise added mass, damping and restoring force (Faltinsen, 1990).

The presence of a body on the surface wave field results in the diffraction of the incident waves and the scattering effect, hence the velocity potential can be written as the sum of incident potential and diffracted potential (Manyanga *et al.*, 2014).

$$F_{ex} = -\rho\, \mathrm{Re}\, i\omega A e^{i\omega t} \iint_{S_B} \left(\phi_I + \phi_D\right)\left(\frac{\partial \phi_j}{\partial n}\right) dS \tag{9}$$

The wave exciting force is divided into two parts (Manyanga *et al.*, 2014). The first part of (9) represents the hydrodynamic part of the Froude-Krylov force. This force, as mentioned earlier, is the dominating part of wave-induced forces acting on any offshore body in the surge, heave and pitch direction. This force is generated by the pressure fields created between the floating body and the incident wave. It is the potential associated with the incident wave in the absence of an offshore structure (Faltinsen, 1990). The second part is the diffraction part or the scattering potential, which describes the interaction between the incident wave and the rigid body. It results from the presence of the body on the wave terrain.

2.5 Velocity Potential

Velocity potential is a consequence of the irrotationality of a flow, and can be defined for a general three-dimensional flow. The velocity potential function satisfies the Laplace equation with a set of boundary conditions; namely, the free surface linear boundary condition, the kinematic bottom boundary condition, the radiation condition which is satisfied at infinity and the body boundary condition applied on the moving body surface in its equilibrium position (Faltinsen, 2005). In addition, the wave velocity potential helps in deriving equations that define the various wave characteristics (such as surface profile, wave celerity, pressure field and particle kinematics).Various analytical methods have been used in the derivation of the velocity potential. For instance, Finnegan *et al.* (2013) used separation of variables and derived an analytical solution for the wave scattering problem for an infinitely long truncated cylinder. In this research, separation of variable method together with linear theory was used in the derivation of the incident velocity potential. From the velocity potential, waves characteristics such as the wave velocity, wave acceleration and wave elevation were derived and used to study such behaviors as effects of the surface waves on the wave amplitude. Furthermore, an investigation of the influence of surface waves on the wavelength, frequency, and period were made.

However, for the study of the effects of surface wave characteristics on the wave exciting force, there was need for appropriate Green functions. Green function, which had an interpretation of a point wave source of unit strength, had been widely used in representing velocity potential such as that developed by Wehausen and Laitone (1960). In addition, Manyanga and Duan, (2012) also developed and used a three dimensional Green functions from Pulsating Sources in a two-layer fluid of a finite depth.

In this study, results in both the surface wave characteristics and internal waves were developed and the results compared. However, most studies adopted the Green function developed by Wehausen and Laitone (1960) in the derivation of the radiation potentials given by equations below,

$$\phi(x,y,z) = \frac{1}{4\pi}\iint_{S_B} f_j(\xi,\eta,\zeta)G(x,y,z,\xi,\eta,\zeta)\,dS \;,\; j = 1,2,\ldots,6 \tag{10}$$

satisfying the Laplace equation,

$$\nabla^2 G(x,y,z,\eta,\xi,\zeta) = \delta(x-\xi)\,\delta(y-\eta)\,\delta(z-\zeta) \tag{11}$$

where,

$$G(P,Q) = \frac{1}{\sqrt{R^2+(y-\eta)^2}} + \frac{1}{(R^2+2h+y+\eta)^2} + 2\int_0^\infty \left(k+\frac{\omega^2}{g}\right)\frac{\cosh k(y+h)\cosh k(\eta+h)}{k\sinh(kh)-\frac{\omega^2}{g}\cosh ky}e^{-kh}J_o(kR)\,dk$$

and

$$P = p(x,y,z)$$

$$Q = q(\xi,\eta,\zeta)$$

$$R = \sqrt{(x-\xi)^2+(z-\zeta)^2} \tag{12}$$

Furthermore, the Green function satisfies not only the Laplace equation but also all the boundary conditions excluding the boundary condition on the body surface.

2.6 Numerical methods

Numerical methods for assessing the sea keeping parameters of offshore structures to predict their motions and loads in waves had been used over the years after the strip theories, which were developed in the early 60's. In addition, there exist practical numerical tools based on three-dimensional analyses that predict linear wave-induced motions and loads on large volume structures at zero Froude number. Nevertheless, strip theory is still used widely (Ghadimi, P., Bandari, P.H. and Rostami, B.A. (2012). This method, however, only provides good results for the vertical responses. In addition, this theory is based on the assumptions of potential flow, slender body and small amplitude motions. Consequently, the theory is computationally efficient, but cannot give accurate hull pressure predictions (Malik *et al.,* 2013). Moreover, for horizontal wave responses, this method needs some modifications to improve the result.

Due to the shortcomings of this theory, researchers have devised better numerical methods to understand the hydrodynamic loads acting on offshore structures. For instance, Yeung (1981) used the eigen function expansion to study forces on a freely floating circular cylinder. The idea of eigen function expansion was extended by Calisal and Sabuncu (1984) to the case of a freely floating circular cylinder and a submerged cylinder resting on the seabed. However, this method has considerable degrees of limitation. For instance, it is insufficient to model structures requiring a very large number of modes, which limits the size of the cross-section for 3-dimensional problems. Other numerical methods such as Boundary Element Method (BEM) and Finite Element Method (FEM) have also been used in the analysis of the hydrodynamics loads of floating structures on the wave terrain. BEM and FEM are efficient in the numerical analysis of hydrodynamics. Unfortunately, they give rise to fully populated matrices. Hence the computation time tends to grow depending on the size of the problem leading to complex computational procedures (Ghadimi *et al.*, 2012). Therefore, to predict properly the interactions between different sections of the floating structure with waves, three-dimension panel method, which is based on potential theory, was developed by Hess and Smith, (1964). Over the years, this method has proved to be more efficient especially in sea-keeping calculations and analysis (Manyanga *et al.,* 2014).

Panel methods attempt to solve the Laplace equation in the fluid domain by distributing sources and dipoles on the body and, in some methods, on the free surface. The surfaces are divided into panels, each of which is associated with a source and dipole distribution of unknown strength, (Malik *et al.,* 2013). The boundary conditions to be applied to the problem are often linearized and they determine either the potential or the normal velocity on each panel. Green's theorem is used to relate the source and dipole distribution strength to the potential and normal velocity on each panel. The number of panels plays a vital role in representing the shape of the body more accurately. Large number of panels ensures more accuracy on the shape of the body, and hence more accurate results. In each panel the potential is taken to be constant. This method has advantages because it helps in reducing the dimensionality of the problem by one and helps in transforming an infinite domain of interest to finite boundaries in which the far field condition is automatically satisfied. Panel method, developed by Hess and Smith (1964), has been used to obtain the radiation potentials and the diffraction potential which are very paramount in the calculation of the diffraction force and the Froude Krylov force (Manyanga *et al.,* 2014). This study applied the same Hess and Smith (1964) method in the determination of the hydrodynamic forces.

2.7 Summary

The body of literature outlined showed clearly that the analysis of hydrodynamic forces especially the wave exciting forces and the effect they had on any offshore body was very important to ocean engineers. It was also evident that many researchers had deliberately failed to analyze these forces with respect to bodies of rectangular cross-section for a fluid of constant density. Therefore, this study investigated and analyzed the forces acting on a rectangular floating barge present on the wave terrain. In addition, due to the limitations of some numerical methods such as BEM, FEM and eigen function expansion the panel method developed by Hess and Smith (1964) was used in this study.

CHAPTER THREE

WAVES CHARACTERISTICS

3.1 Introduction

When working on offshore rigid or floating bodies, it is important to understand how these structures responded to waves. Ocean surface waves are known to cause periodic loads on all man-made structures in the sea, whether these structures are fixed, floating or sailing and on the surface or deeper in the sea. To understand these loads, a good understanding of the physics of water waves is necessary. Analysis of incoming waves is especially of great importance to ocean engineers and designers. Furthermore, these waves have essential characteristics, which need proper understanding. These characteristics include; incident wave potential, incident wave elevation, wave velocity and acceleration. For instance, the incidence wave velocity potentials are very important for the analysis of the plane progressive wave. In this study analysis of vertical wave elevation, velocity and acceleration, as a result of incident wave potential was done. The influence of water depth on the wave characteristics was also investigated. The incident velocity potential was solved by separation of variables where appropriate boundary conditions were imposed.

3.2 Mathematical formulations

A Cartesian system was adopted, $o-xyz$, fixed on the fluid with oy opposing the direction of gravity and $o-xz$ lying on the undisturbed free surface. The plane $y=0$ denotes the undisturbed free surface. The surface displacements of the water from the mean was given by $y=\eta(x,y,z)$ which is the wave elevation. The water depth h , was measured between the sea bed $(y=-h)$ and the still water level $(y=0)$. Surface water waves were considered. The amplitude of the wave which is the distance between the still water level and the wave crest was assumed to be small than it wavelength λ . The wave was assumed to be periodic so that the wave period was taken as the time required by one wave to pass a particular point. The problem was governed by certain boundary conditions.

3.3 Governing equations

Since the fluid is incompressible, the divergence of its velocity field is zero everywhere,

$$\vec{\nabla} \bullet \vec{V} = 0 \tag{13}$$

The flow in this study is assumed irrotational. Then, there exist a velocity potential $\phi(x, y, z)$ such that the velocity components in the $x, y, z - axis$ is given as;

$$\frac{\partial \phi}{\partial x} = u, \frac{\partial \phi}{\partial y} = v, \frac{\partial \phi}{\partial z} = w \tag{14}$$

Substituting equation (13) into equation (14) the velocity potential satisfies Laplace equation

$$\nabla^2 \phi = 0 \tag{15}$$

$$\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial v^2} + \frac{\partial^2 \phi}{\partial w^2} = 0 \tag{16}$$

The velocity potential is subject to kinematic and dynamic free surface boundary conditions.

The boundary conditions for this case are as follows:

i) The sea bottom condition, that is, the velocity components in the y-direction must go to zero at the sea bottom.

$$v = \frac{\partial \phi}{\partial y} = 0, \quad y = -h \tag{17}$$

ii) The dynamic free-surface condition which simply means that the fluid pressure at the free surface is equal to the atmospheric pressure.

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right] + \frac{p}{\rho} + gy = 0 \tag{18}$$

Since in this study linear theory (velocity potential is proportional to the wave amplitude) was used then the high powers in equation (18) were ignored, and also by use sea bottom condition equation (18) at the sea bed reduces to:

$$\left(\frac{\partial \phi}{\partial t}\right)_{y=-h} + g\eta = 0 \tag{19}$$

where,

$$y = \eta\left(x, z, t\right) \tag{20}$$

Introducing a function $F\left(x, y, z, t\right)$ then the substantial derivative of the function F, defined by equation (21), is given by equation (22). Substantial derivative expresses the time rate of change when we follow a fluid particle as it moves in space.

$$F\left(x, y, z, t\right) = y - \eta(x, z, t) = 0 \tag{21}$$

Equation (21) states that a fluid particle on the free surface always remain on the free surface. The material derivative of equation (21) becomes,

$$\frac{DF}{Dt} = \left(\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + w\frac{\partial F}{\partial z} - y\right)_{y=0} = 0 \tag{22}$$

Applying the same material derivative on equation (20) we obtain,

$$\frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)_{y=\eta} + \frac{\partial \eta}{\partial z}\left(\frac{\partial \phi}{\partial z}\right)_{y=\eta} = \left(\frac{\partial \phi}{\partial y}\right)_{y=\eta} \tag{23}$$

But from equation (19), we have,

$$\left(\frac{\partial \phi}{\partial t}\right)_{y=0} + g\eta = 0 \tag{24}$$

However, from the free surface condition equation (23) reduces to,

$$\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{25}$$

From equation (24) and equation (25), we have,

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial y} = 0,\ y = 0 \tag{*}$$

When the velocity potential is oscillating harmonically in time with circular frequency ω, then equation (*) can be written as

$$-\omega^2\phi + g\frac{\partial\phi}{\partial y} = 0 \text{, at y=0} \tag{26}$$

3.3 Solution for the velocity potential

Assuming that the free surface wave elevation η take the form given by equation (27) below

$$\eta = ae^{i(k_1 x\cos\theta + k_2 z\sin\theta - \omega t)} \tag{27}$$

The aim is to find the velocity potential ϕ satisfying the above boundary conditions. Assuming that the velocity potential can be represented as product of functions where each function depend on only one independent function then equation (16) is solved by the separation of variables. Let the incident wave velocity potential takes the form given by equation (28)

$$\phi = f(y)e^{i(k_1 x\cos\theta + k_2 z\sin\theta - \omega t)} \tag{28}$$

Finding the second order derivative of equation (28) we have;

$$\nabla^2\phi = \frac{\partial^2 f}{\partial y^2} - k^2 f(y) = 0 \tag{29}$$

where,

$$k = \sqrt{\left(k_1^2 + k_2^2\right)} \tag{30}$$

The solution of equation (29) is given by equation (31) below,

$$y = Ae^{ky} + Be^{-ky} \tag{31}$$

Substituting equation (31) into equation (28), we have,

$$\phi = \left(Ae^{ky} + Be^{-ky}\right)\left(e^{i(k_1 x\cos\theta + k_2 z\sin\theta - \omega t)}\right) \tag{32}$$

From the second order derivative of equation (32) and by use of equation (16) then it follows that;

$$Ae^{-kh} - Be^{kh} = o$$
$$Ae^{-kh} = Be^{kh} \quad (33)$$

Substituting equation (32) in equation (26), we have

$$\left(\omega^2 - gk\right)A + \left(\omega^2 + gk\right)B = 0 \quad (34)$$

Writing equation (33) and equation (34) in matrix form, we have

$$\begin{vmatrix} e^{-hk} & -e^{hk} \\ \omega^2 - gk & \omega^2 + gk \end{vmatrix} = 0 \quad (35)$$

Solving equation (35) and making ω^2 the subject of the formula we obtain,

$$\omega^2 = gk\left(\frac{e^{kh} - e^{-kh}}{e^{kh} + e^{-kh}}\right) = gk \tanh kh \quad (36)$$

But the wave celerity, c is given by equation (37) below,

$$c = \frac{\omega}{k} \quad (37)$$

Hence applying equation (36) into equation (37) we have

$$c = \sqrt{\left(\frac{g}{k}\tanh hk\right)} \quad (38)$$

Let $Ae^{-kh} = Be^{kh} = \frac{D}{2}$ (39)

Applying equation (39) on equation (32) we obtain,

$$\phi = \frac{1}{2}D\left\{e^{k(h+y)} + e^{-k(h+y)}\right\}\left(e^{i(kx\cos\theta + kz\sin\theta - \omega t)}\right) \quad (40)$$

But considering the definition of hyperbolic function then equation (40) can be reduced to equation (41) below,

$$\phi = \left(D \cosh k(h + y) \right) e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \tag{41}$$

However equation (24) can be rewritten in the form of equation (42) below.

$$\left(\frac{\partial \phi}{\partial t} \right)_{y=0} = -\eta g \tag{42}$$

Differentiating equation (41) with respect to time and applying equation (42) we obtain

$$\eta = \frac{i\omega}{g} D \cosh kh \left(e^{i(kx\cos\theta + kz\sin\theta - wt)} \right) = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right) \tag{43}$$

Equation (43) can be reduced into equation (44),

$$\eta = a e^{i(kx\cos\theta + ky\sin\theta - \omega t)} \tag{44}$$

Then from equation (36) we have,

$$\frac{\omega^2}{g} \cosh kh = k \sinh kh \tag{45}$$

$$D = -i \frac{ag}{\omega \cosh kh} = -i \frac{a\omega}{k} \frac{1}{\sinh kh} \tag{46}$$

Substituting equation (46) into equation (41) the velocity potential due to incidence waves is given by equation (47) below,

$$\phi = -i \frac{a\omega}{k} \frac{\cosh k(h + y)}{\sinh kh} e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \tag{47}$$

However, by Euler's formula,

$$e^{i\alpha t} = \cos\alpha t + i \sin\alpha t \tag{48}$$

Hence, using equation (48), equation (34) becomes;

$$\phi = -i \frac{a\omega}{k} \frac{\cosh k(y + h)}{\sinh kh} \left[\cos\left(kx\cos\theta + kz\sin\theta - wt\right) + i \sin\left(kx\cos\theta + kz\sin\theta - \omega t\right) \right]$$

$$\phi = \frac{a\omega}{k} \frac{\cosh k(y + h)}{\sinh kh} \sin\left(kx\cos\theta + kz\sin\theta - \omega t\right) \tag{49}$$

Equation (49) is the velocity potential satisfying the boundary conditions aforementioned.

3.4 Wave elevation

Wave elevation was an important aspect of the wave that needed investigation this is because it was used in the determination of the vertical relative motion of any floating structure with respect to the undisturbed wave surface. Vertical motion was a very significant aspect that coastal engineers should always put into consideration. For instance in ships, vertical motions were and are still used to predict damages that might occur due to slamming and water in the deck. From equation (49) and the boundary condition on equation (42), the wave elevation is given by equation (50) below.

$$\frac{\partial \phi}{\partial t} = \eta = \frac{a\omega^2 \cosh k\left(y+h\right)}{k \sinh kh} \cos\left(kx\cos\theta + kz\sin\theta - \omega t\right) \qquad (50)$$

This is a cosine wave profile.

3.5 Wave velocity

Wave velocity is an important aspect that needs to be considered by researchers in the field of hydrodynamics. Vertical velocity has been associated with high vertically induced motions Vasquez et al., 2011). Wave velocity was derived directly from the velocity potential. This was possible due to the irrotationality condition of the fluid. Differentiating equation (49) partially along each axis we obtain;

$$u = \frac{\partial \phi}{\partial x} = \frac{a\omega \cos\theta \cosh k(y+h)}{\sinh kh} \cos\left(kx\cos\theta + kz\sin\theta - \omega t\right)$$

$$v = \frac{\partial \phi}{\partial y} = \frac{a\omega \sinh k(y+h)}{\sinh kh} \sin\left(kx\cos\theta + kz\sin\theta - \omega t\right) \qquad (51)$$

$$w = \frac{\partial \phi}{\partial z} = \frac{aw \sin\theta \cosh k\left(y+h\right)}{\sinh kh} \cos\left(kx\cos\theta + kz\sin\theta - \omega t\right)$$

Equation (51) represents the of the wave velocity in 3-dimension. These velocity equations express the local fluid velocities at any distance *(y+h)* above the impermeable bottom. Furthermore, they are periodic. The hyperbolic functions give the exponential decay of the magnitude of the velocity components in respect to increase of distance below the free surface. This means that as the depth of the fluid increases the wave motions reduces. This is in accordance with shallow water effect.

3.6 Wave acceleration

Wave acceleration is an important aspect since acceleration is associated with the forces. This is in accordance with Newton second Law of motion. Acceleration represents the change that a wave undergo with respect to time. This therefore means that differentiating equation (51) with respect to time we obtain the acceleration in 3-dimension given by equation (52) as follows,

$$\begin{aligned} u_t &= \frac{a\omega^2 \cos\theta \cosh k(y+h)}{\sinh kh} \sin(kx\cos\theta + kz\sin\theta - \omega t) \\ v_t &= -\frac{a\omega^2 \cosh k(y+h)}{\sinh kh} \cos(kx\cos\theta + kz\sin\theta - \omega t) \\ w_t &= \frac{a\omega^2 \sin^2\theta \cosh k(y+h)}{\sinh kh} \sin(kx\cos\theta + kz\sin\theta - \omega t) \end{aligned} \tag{52}$$

The negative sign in one of the above equations shows that the direction of the wave is changing with respect to the origin. The acceleration of the wave is linearly proportional to that of the body. Vertical acceleration in particular between the body and the waves is responsible for determining the cargo weight in a ship and has also been associated to sea sickness.

3.7 Results and discussion

When the above equations were coded using Fortran software they led to the development of the following graphs.

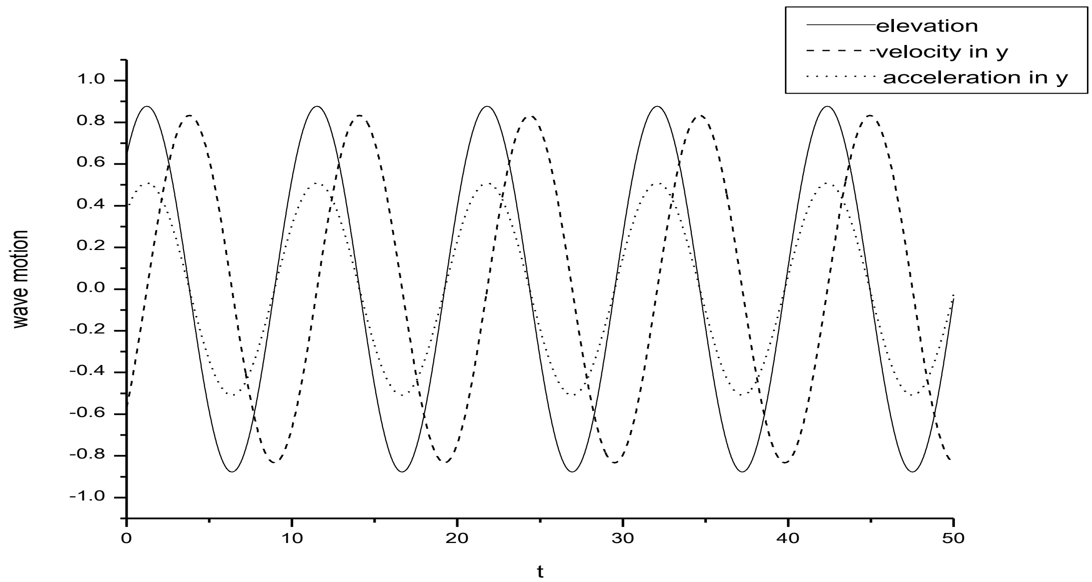

Figure 1: Graph showing the relationship between wave elevation and vertical velocity and acceleration.

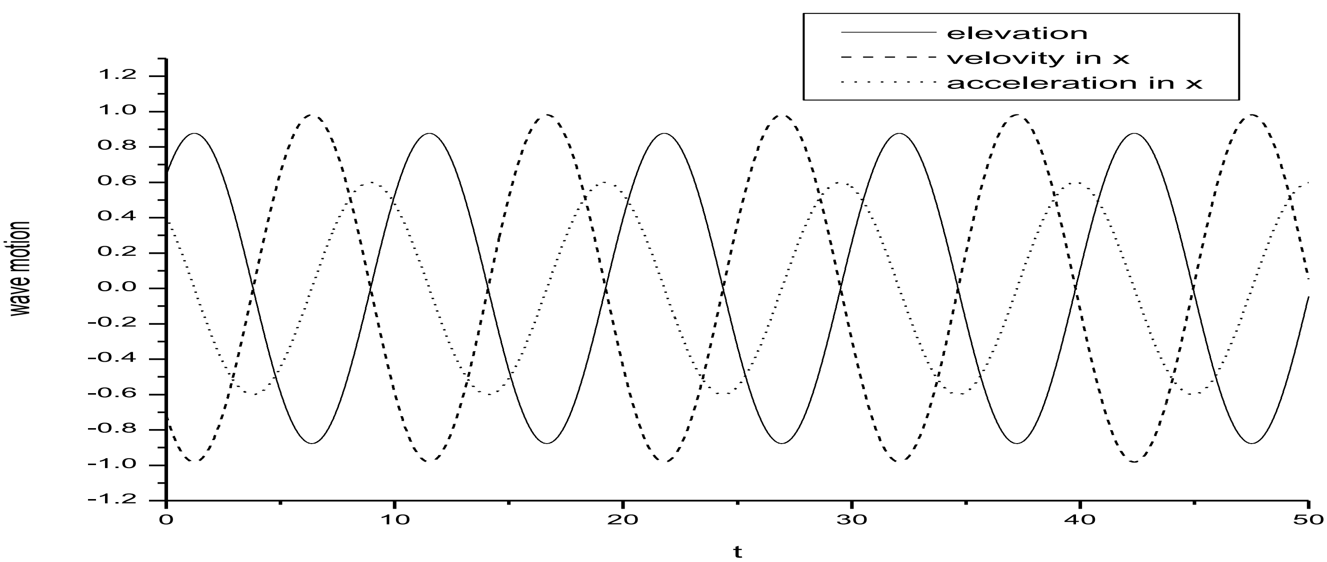

Figure 2: **Graph showing the relationship between wave elevation, velocity and acceleration on the x direction**

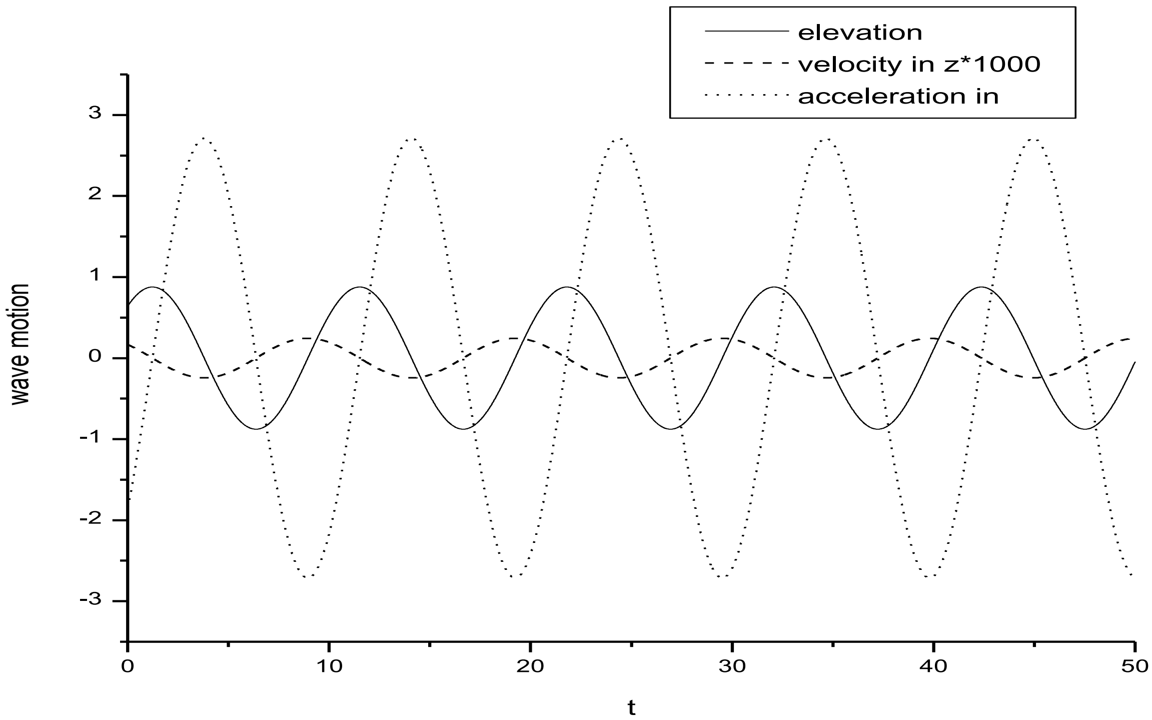

Figure 3: Graph showing the relationship between wave elevation, velocity and acceleration in z direction

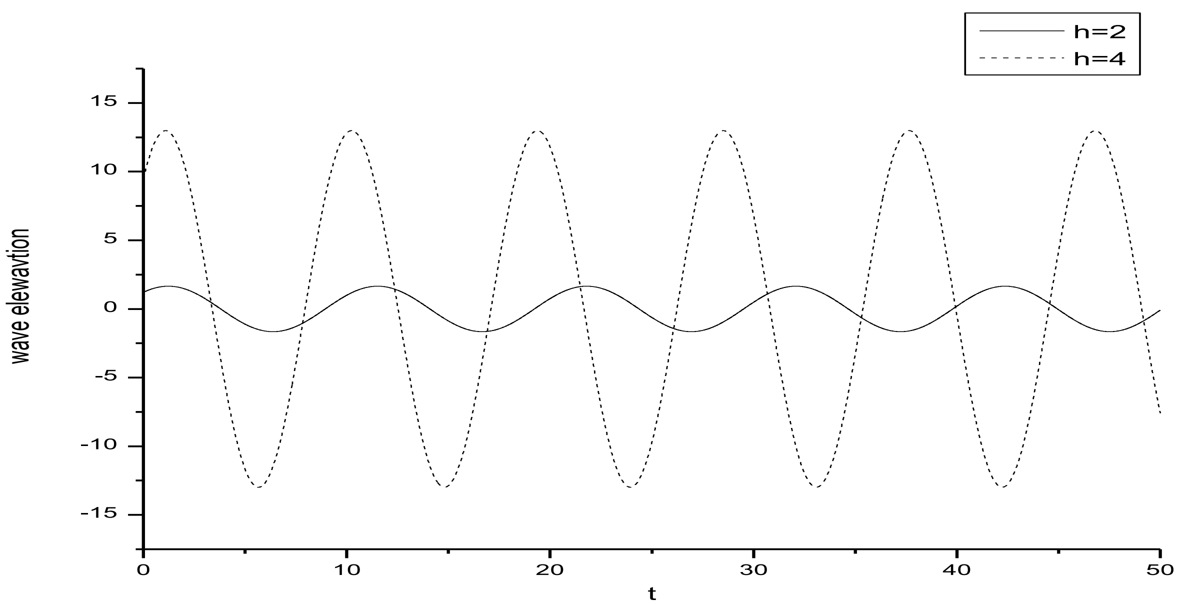

Figure 4: Graph showing the wave elevation at different water depth from the mean surface.

Figure 1 shows the relationship between wave elevation, velocity and acceleration in the vertical direction. From the figure it was observed that the wave's vertical velocity and acceleration were out of phase by 90°. However, the period of this wave remained constant. This was in agreement with the results obtained by Manyanga, O. D., and Wen-Yang (2011) in their study of Three Dimension Internal Waves due to Pulsating Sources and Oscillation of floating Bodies. It was also evident that the amplitude of the wave velocity motion was larger than that of acceleration. Moreover, wave energy was directly proportional to the square of the amplitude of the wave. Furthermore, from the oscillatory wave structures we observed that when the velocity amplitude was at a maximum, the acceleration amplitude was at a minimum.

Waves with high vertical velocity induced a lot of motion on a body especially if the body was too close to the origin of the wave. Very high velocity impacts between offshore structures and the waves were associated with slamming. However, since the impact occurs over a small period of time, the gravity accelerations was assumed negligible relative to the impact induced accelerations (Baarholm and Stansberge, 2005).

When an offshore structure came into contact with these waves, the added mass and damping coefficients corresponded to components that were in phase with the accelerations and velocity of the offshore structure. However if the added mass and the damping coefficients of the body was zero then the oncoming waves is predicted to be equal to the outgoing waves. In summary, acceleration and velocity of the structure were proportional to the acceleration and velocity of the surface waves respectively.

Figure 2 shows the relation between elevation, velocity and acceleration in the direction of wave propagation. On the other hand, comparing figure 2 and figure 1, it was found that the velocity in x direction and the elevation had a very big difference. For instance, when wave elevation motion amplitude was at maximum, the velocity amplitude was at minimum, this lead to the formation of a standing wave. This showed that the horizontal force generated by the waves did not induce a lot of motions on the floating body. This study concluded that the horizontal characteristic of wave did not lead to high-induced motion and hence the effect they had on a floating body were not catastrophic.

It is clearly depicted from figure 3, that the velocity of the wave motion in the z -axis is negligible. The amplitude of the entire wave characteristic as far as this axis is concerned was so small contrary to the other axes.

Figure 4, shows that the depth of the fluid greatly affects the number of oscillations. As the depth increased, the frequency also increased. Furthermore, increase in depth led to an increase on the wavelength though at a slower rate. In addition, another interesting observation was that the horizontal particle displacement was large when the depth was small. The numerical results were as expected for the shallow water effects on the wave force.

In summary, it was observed that most waves' characteristics that is; celerity, height, length, surface profile, water particles velocity and acceleration were highly affected by change in depth. However, the wave period remained constant.

CHAPTER FOUR

HYDRODYNAMIC FORCES

4.1 Introduction

The problem of determining the hydrodynamic forces acting on a rigid floating body present on the wave terrain has been studied extensively over the years, though not much has been done on rectangular floating barge in incompressible fluid. Most researchers who embarked on the study of these forces in respect to rectangular floating bodies did so with the assumptions that the density of the fluid is not constant due to factors such as salinity and temperature change on the ocean (Manyanga *et al.*, 2014; Nguyen' and Yeng, 2010). The hydrodynamic forces are associated with surface waves generated by; the interaction between winds and the sea surface, moving vessels, hurricanes, seismic disturbances or the gravitational pull of the sun and the moon. However in this study the research was carried out for fluids of constant density. The hydrodynamic forces considered in this study were the Froude Krylov force and the diffraction forces. The study of the aforementioned forces is of great help to researchers in the analysis and prediction of wave exciting forces.

4.2 Mathematical Formulation

Three-dimensional problem concerning the hydrodynamic behavior of a rectangular floating barge in the coastal marine environment was considered. The barge measurements were taken as follows; Length, L = 2.25 m; Breadth, B = 2.25 m; and Draught, T = 1.00 m. The floating body was taken to be at a distance H from the free surface and was assumed to have zero forward speed. In addition, the sea bed was taken to be at a distance h from the free surface. This problem was analyzed in response to oncoming regular waves with small amplitude A as compared to the wavelength λ . The sea environment consisted of a water layer of finite depth bounded above by the free surface and below by a rigid bottom. A fixed Cartesian coordinate (x, y, z) was introduced with its origin at some point on the mean water level which was taken to coincide with the centre of floatation of the rigid floating body, and the $y-axis$ pointing upward through the centre of gravity of the body as shown in figure 5. The body surface of the barge was defined by a set of points which were presumably exactly on the surface. These points were associated in a group of four to form quadrilateral surface elements. Each point on the surface was used in the formation of four quadrilaterals, and thus the total number of quadrilateral was approximately the same as the number of points used to define the body surface. The integration of pressure on each panel was carried out by the method of Hess and Smith (1964), 192 Panel were used. In order to evaluate the physical problem, the study was based on the assumptions that the fluid is homogeneous, inviscid and incompressible and that the flow is irrotational.

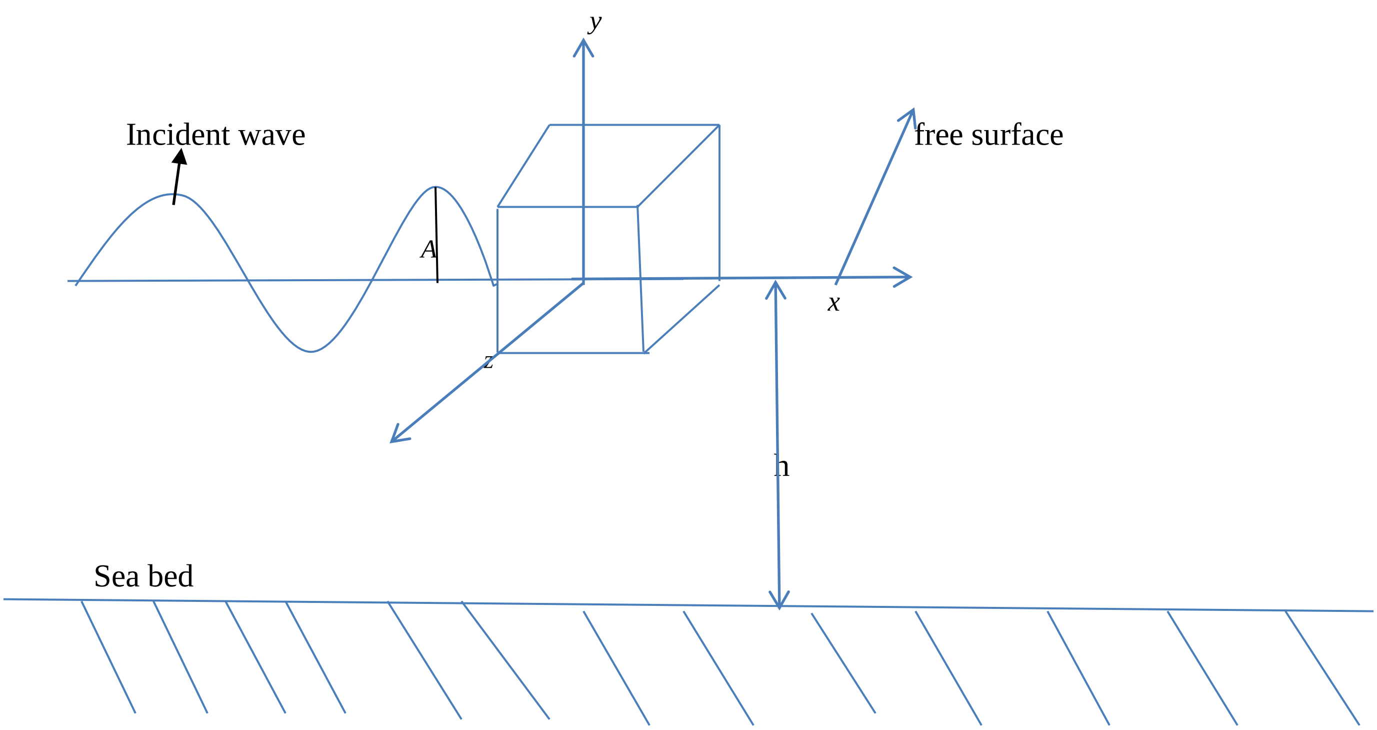

Figure 5: Schematic diagram representing rectangular floating body on the incident wave field.

4.2.1 Diffraction problem

The boundary integral method together with the Greens Function was used in getting the velocity potentials on the wetted surface of the rectangular floating barge. The wetted surface of the rectangular floating barge was divided into small rectangular panels capable of representing a curved surface so as to reduce the deflection of wave due to the edges of the body and avoiding "leakage" gaps, and small enough so as to help in assuming that the source strength and the fluid pressure (velocity potential) are constant in each panel. This approach was carried out as follows: the body surface was defined by a set of points which were presumably exactly on the surface.

These points were associated in a group of four to form a quadrilateral surface element. Each point on the surface was used in the formation of four quadrilaterals, and thus the total numbers of quadrilateral were approximately the same as the number of points used to define the body surface. The integration on each panel was carried out by the method of Hess and Smith (1964). In this case, the velocity potential was assumed to be constant over the panel. The boundary integral equations for radiation potentials were derived for the rectangular floating barge. These radiation potentials were used to solve diffraction forces and the Froude-Krylov forces for the rectangular floating barge on fluid of finite depth. Consequently, a Fortran code was written to help in getting the wave exciting forces. The data obtained was imported to Origin software where graphs showing the magnitude of heave and surge wave exciting forces on a rectangular floating barge were obtained. The results obtained were compared to other results that had been obtained by Manyanga and Duan (2014) and Endo (1987).

From the assumptions of linear water wave theory, the total velocity potential can be divided into a known incident potential and an unknown diffracted potential (Manyanga *et al.*, 2014)

$$\phi = \phi_I + \phi_D \tag{53}$$

The diffraction potential must satisfy the following boundary conditions and governing equations.

The Laplace equation; $$\nabla^2 \phi = 0 \tag{54}$$

The sea bottom boundary conditions; $$\frac{\partial \phi_D}{\partial y} = -\frac{\partial \phi_I}{\partial y}, Seabed \tag{55}$$

The kinematic free surface boundary conditions; $$\frac{\partial \eta_D}{\partial t} = \frac{\partial \phi_D}{\partial y}, y = 0 \tag{56}$$

The body surface boundary condition; $\frac{\partial \phi_D}{\partial t} = -g\eta_D, y = 0$ (57)

Radiation boundary condition; $\nabla \phi \to 0, t \to \infty, s_\infty$ (58)

It is worth noting that both the ϕ_D and $\frac{\partial \phi_D}{\partial t}$ are bounded uniformly on s_∞. Furthermore, since the free surface boundary conditions is second order in time, then the velocity of the body disturbances decays to undisturbed flow away from the body, then it follows that,

$$\phi_D \to 0, t \to -\infty \tag{59}$$

$$\frac{\partial \phi_D}{\partial t} \to 0, t \to -\infty \tag{60}$$

4.2.2 Integral equations

In the derivation of the integral equations for the diffraction potential, the Wehausen and Laitone (1960) time dependent Green's function given by equation (61) below is used

$$\phi(x, y, z) = \frac{1}{4\pi} \iint_{s_B} f_{S_B}(\xi, \eta, \zeta) G(x, y, z, \xi, \eta, \zeta). \tag{61}$$

This green function in equation (61) satisfies the Laplace equation (62)

$$\nabla^2 G(x, y, z, \xi, \eta, \zeta) = \delta(x - \xi)\delta(y - \eta)\delta(z - \zeta) \tag{62}$$

where $G(x, y, z,, \xi, \eta, \zeta)$ corresponded physically to the potential of the oscillatory source. Equation (61) and equation (62) shows the transformation of a function $f(x, y, z)$ to the function $f(\xi, \eta, \zeta)$. In addition the green function is subject to the same boundary conditions as the incident velocity potential that is, the conditions on the free surface and at the sea bottom as well as the radiation conditions. The green function at a source and a field point can be described by equation (63) below,

$$G(P,Q) = \frac{1}{\sqrt{R^2 + (y-\eta)^2}} + \frac{1}{(R^2 + 2h + y + \eta)^2} + 2\int_0^{\infty}\left(k + \frac{\omega^2}{g}\right)\frac{\cosh k(y+h)\cosh k(\eta+h)}{k\sinh(kh) - \frac{\omega^2}{g}\cosh ky}e^{-kh}J_o(kR)\,dk \tag{63}$$

where

$$P = p(x, y, z)$$

$$Q = q(\xi, \eta, \zeta)$$

$$R = \sqrt{(x-\xi)^2 + (z-\zeta)^2}$$

Applying the Green's second identity below,

$$\iiint_V \left(\phi_D \nabla^2 G - G\nabla^2 \phi_D\right)dv = \iint_s \left(\phi_D \frac{\partial G}{\partial n} - G\frac{\partial \phi_D}{\partial n}\right)ds = 0 \tag{64}$$

into equation (63) we obtain the integral equation (65) below,

$$-2\pi\phi_D(P) + \iint_S \phi_D \frac{\partial G(P,Q)}{\partial n_Q}dS_Q = \iint_S G(P,Q)\frac{\partial \phi_D}{\partial n_Q}(Q)\,dS_Q \tag{65}$$

Equation (65) is the Fredholm equation of the second kind for the values of the potential on the body surface, the entire surface of integration will be described by the sum of all its parts;

$$S = S_B \cup S_F \cup S_{-\infty} \cup S_{\infty} \tag{66}$$

4.2.3 Wave exciting force

The wave exciting force is a combination of the Froude Krylov and the diffraction force.

These forces are heavily related to the velocity potential given by equation (49) and equation (61)

$$\phi = \phi_I + \phi_D \tag{67}$$

Where, ϕ_I, ϕ_D are the velocity potential of the incident wave and diffraction wave respectively. In addition due to the assumption of small amplitude motions then these velocity potentials must satisfy the following boundary conditions as aforementioned.

i) Free surface boundary conditions

$$\frac{\partial \phi}{\partial y} = \frac{\omega^2}{g}\phi \text{, at } y = 0 \tag{68}$$

ii) Bottom boundary condition

$$\frac{\partial \phi}{\partial y} = 0, \text{ at } y = -h \tag{69}$$

iii) $$\frac{\partial \phi_I}{\partial n} = \vec{n} \bullet \vec{\nabla} \phi_I \tag{70}$$

and, $$\frac{\partial \phi_I}{\partial n} = -\frac{\partial \phi_D}{\partial n} \tag{71}$$

iv) The radiation boundary condition

$$\lim_{R \to \infty} \sqrt{R}\left(\frac{\partial \phi}{\partial n} - ik\phi\right) = 0 \tag{72}$$

Suppose that the diffraction potential is known and from equation (49) the incident velocity potential is given by:

$$\phi_I = \frac{ag}{\omega}\frac{\cosh k(y+h)}{\cosh kh} e^{ik(x\cos\theta + z\sin\theta - \omega t)} \tag{73}$$

This implies that the total velocity potential given by equation (74) below is known,

$$\phi = \phi_I + \phi_D \tag{74}$$

Consequently, the dynamic pressure can be derived from equation (4) that is;

$$p + \rho\frac{\partial \phi}{\partial t} + \frac{\rho}{2}(\nabla \phi)^2 + \rho g y = c \tag{75}$$

Applying linear theory, equation (75) reduces to

$$p = -\rho\frac{\partial \phi}{\partial t} = -\rho\frac{\partial \phi_I}{\partial t} - \rho\frac{\partial \phi_D}{\partial t} \tag{76}$$

The hydrodynamic forces acting on the body were found by integrating the pressure (equation (76)) over the body surface. In general each mode of motion can induce forces in all the six direction, therefore, the dynamic force and the moments acting on the body can be defined as;

$$\vec{F} = \iint_S P \, \vec{n}_j dS, \; j = 1,2,3 \tag{77}$$

$$\vec{M}_j = \iint P(\vec{r} \times \vec{n})_{j-3} \, \mathrm{dS}, \; j = 4,5,6 \tag{78}$$

Where p in the above equations is the fluid pressure and can be derived through Bernoulli's principle.

Substituting equation (76) into equation (77) the net force is obtained as

$$\vec{F} = \iint_S \rho \frac{\partial \phi_I}{\partial t} \vec{n} \, dS + \iint_S \rho \frac{\partial \phi_D}{\partial t} \vec{n} \, dS \tag{79}$$

Equation (79) is the wave exciting force which is a combination of the Froude Krylov force and the diffraction force.

From equation (79) diffraction force is obtained as;

$$\vec{F}_D = \iint_S \rho \frac{\partial \phi_D}{\partial t} \vec{n} \, dS \tag{80}$$

but

$$\vec{n} = \frac{\partial \phi_j}{\partial n} \tag{81}$$

Then substituting equation (81) to equation (80) the diffraction forces can be expressed as shown in equation (82) below,

$$\vec{F}_D = \iint_S \rho \frac{\partial \phi_D}{\partial t} \frac{\partial \phi_j}{\partial n} dS \tag{82}$$

But

$$\frac{\delta \phi_D}{\delta t} = i\omega \phi_D$$

Equation (82) can be rewritten as

$$\overrightarrow{F}_D = i\rho\omega \iint_S \phi_D \frac{\partial \phi_j}{\partial n} dS \tag{83}$$

or

$$\overrightarrow{F}_D = i\rho\omega \iint_S \phi_j \frac{\partial \phi_D}{\partial n} dS \tag{84}$$

but the incident velocity potential and the diffraction potential are related by equation (71) that is;

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n}$$

Therefore, applying equation (71) on equation (84) the diffraction force is obtained as follows,

$$\overrightarrow{F_D} = -i\rho\omega \iint_S \phi_j \frac{\partial \phi_I}{\partial n} dS, \ \text{j} = 1, 2, \ldots, 6 \tag{85}$$

We now derive the Froude Krylov force which is related to the incoming wave potential.

$$\overrightarrow{F}_I = -\rho \iint_S \frac{\partial \phi_I}{\partial t} \left(-\overrightarrow{n} \right) dS \tag{86}$$

but from equation (68) we have,

$$\frac{\partial \phi_I}{\partial t} = \frac{\omega^2 \phi_I}{g} = i\omega \phi_I \tag{87}$$

Consequently, substituting equation (87) into equation (86) we obtain;

$$\begin{aligned} \overrightarrow{F}_I &= \iint_S i\rho\omega \phi_I \overrightarrow{n} dS \\ &= i\omega\rho \iint_S \phi_I \frac{\partial \phi_j}{\partial n} dS \end{aligned} \tag{88}$$

However, wave exciting forces acting on a body is a combination of the Froude Krylov force and the diffraction force, combining equation (85) and equation (88) then the wave exciting force becomes;

$$\overrightarrow{F}_I + \overrightarrow{F}_D = -i\omega\rho \iint_S (\phi_I + \phi_D) n_j dS \tag{89}$$

$$\overrightarrow{F}_E = i\rho\omega \iint_S \left(\phi_I \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_I}{\partial n} \right) dS \tag{90}$$

4.2.4 Mathematical scheme

Rewriting equation (49) and equation (90) respectively;

$$\phi_I = \frac{g}{\omega^2} \frac{\cosh k(y+h)}{\cosh kh} e^{ik(x\cos\theta + z\sin\theta - \omega t)}$$

NOTE

Unit normal vector

$$\frac{\partial \phi_j}{\partial n} = n_j, \text{ j} = 1, 2, \ldots, 6 \tag{91}$$

where,

$$n_1 = n_x, n_2 = n_z, n_3 = n_y \tag{92}$$

$$D = \begin{vmatrix} i & j & k \\ x & z & y \\ n_x & n_z & n_y \end{vmatrix} \tag{93}$$

$$D = \left(zn_y - yn_z \right) i - \left(xn_y - yn_x \right) j + \left(xn_z - zn_x \right) k$$

Then from equation (93) we have;

$$n_4 = zn_y - yn_z, n_5 = yn_x - xn_y, n_6 = xn_z - zn_x \tag{94}$$

Therefore,

$$\iint_S \phi_I \frac{\partial \phi_j}{\partial n} ds = \iint_S \phi_I n_j ds \tag{95}$$

For heave motions equation (95) becomes

$$\iint_S \phi_I \frac{\partial \phi_3}{\partial n} ds = \iint_S \phi_I n_y ds \tag{96}$$

$$\frac{\partial \phi_I}{\partial n} = \vec{n} \bullet \vec{\nabla} \phi_I \tag{97}$$

Where, the unit normal is given by the equation (98) below;

$$\vec{n} = n_1 \vec{i} + n_2 \vec{j} + n_3 \vec{k} \tag{98}$$

$$= n_x \vec{i} + n_z \vec{j} + n_y \vec{k}$$

$$\vec{\nabla} \phi_I = \frac{\partial \phi_I}{\partial x} \vec{i} + \frac{\partial \phi_I}{\partial z} \vec{j} + \frac{\partial \phi_I}{\partial y} \vec{k} \tag{99}$$

The velocity potential normal to the surface is given by,

$$\frac{\partial \phi_I}{\partial n} = \left(n_x \vec{i} + n_z \vec{j} + n_y \vec{k} \right) \bullet \left(\frac{\partial \phi_I}{\partial x} \vec{i} + \frac{\partial \phi_I}{\partial z} \vec{j} + \frac{\partial \phi_I}{\partial y} \vec{k} \right) \tag{100}$$

$$= n_x \frac{\partial \phi_I}{\partial x} + n_z \frac{\partial \phi_I}{\partial z} + n_y \frac{\partial \phi_I}{\partial y}$$

$$\iint_s \phi_j \frac{\partial \phi_I}{\partial n} ds = \phi_j \iint_s \left(n_x \frac{\partial \phi_I}{\partial x} + n_z \frac{\partial \phi_I}{\partial z} + n_y \frac{\partial \phi_I}{\partial y} \right) ds \tag{101}$$

4.3 Results and discussion

For the analysis of the hydrodynamics loads a Fortran code was written which read to the formulation of the graphs below.

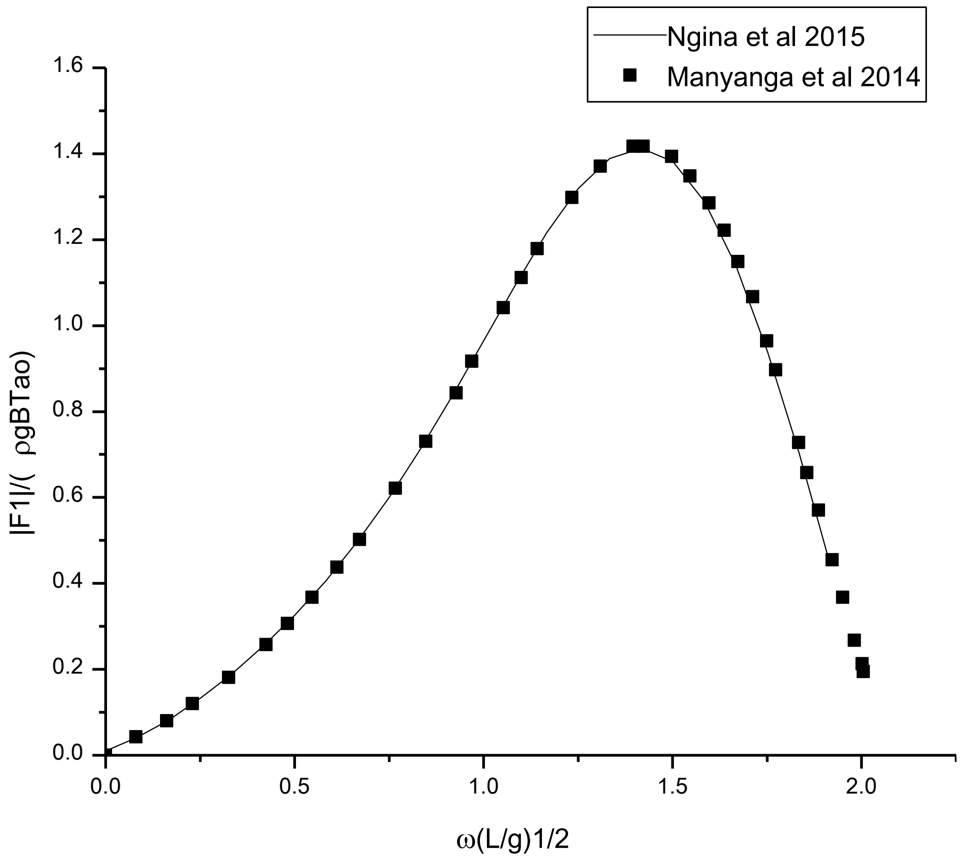

Figure 6: A graph showing the magnitude of surge wave exciting forces

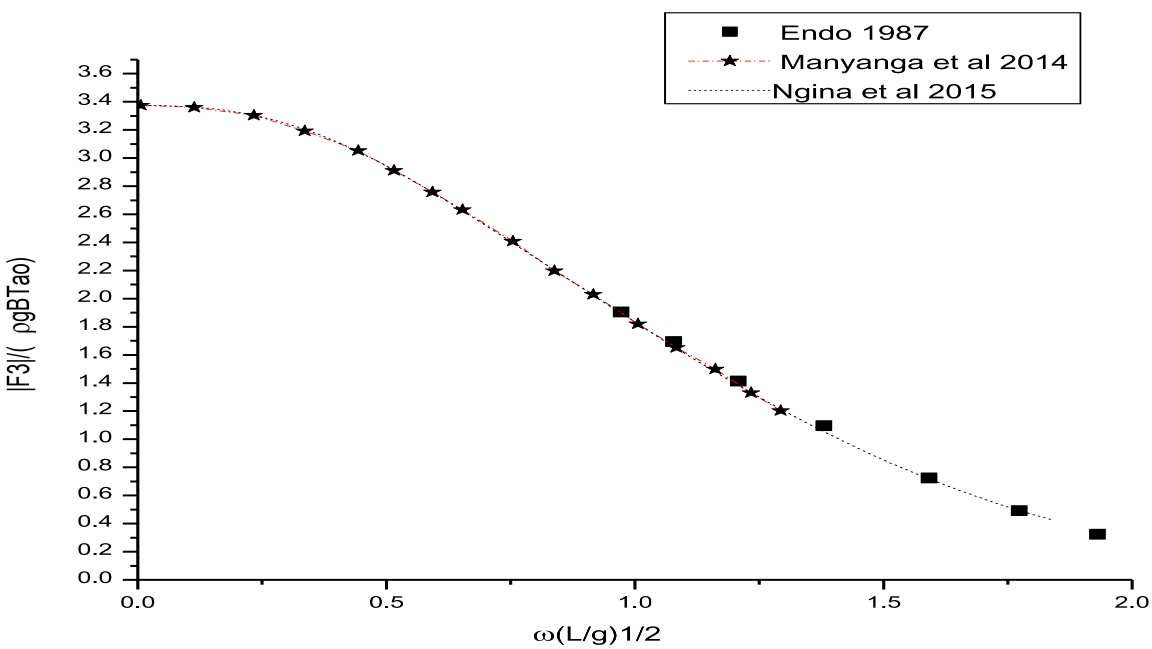

Figure 7: Graph showing the magnitude of heave wave exciting force

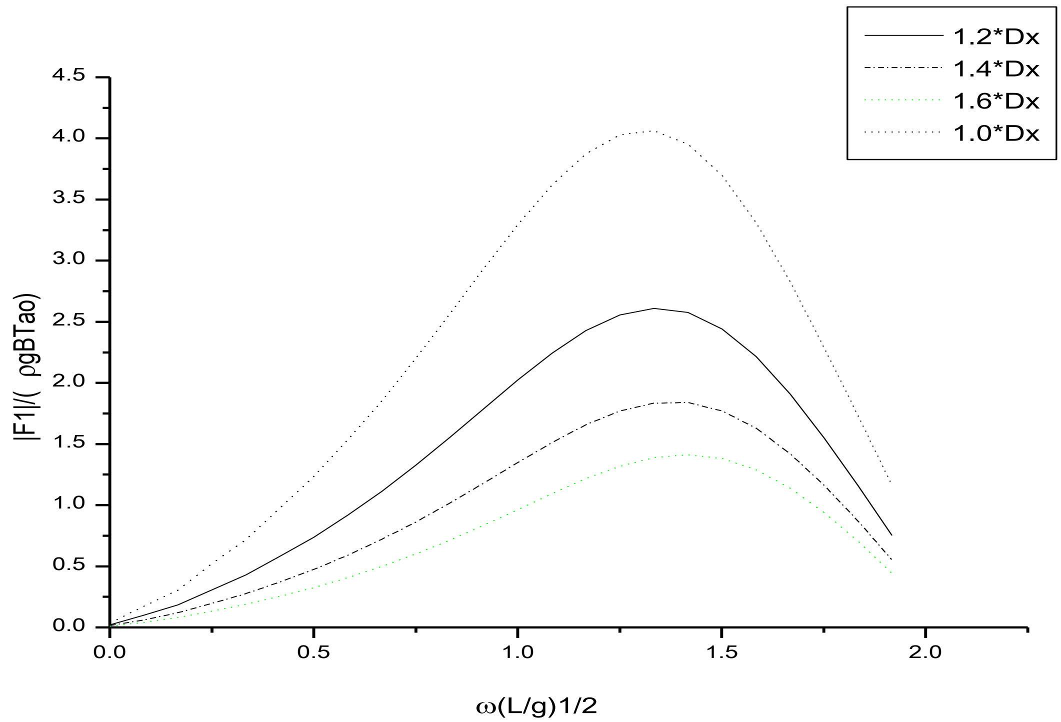

Figure 8: Graph showing the magnitude of surge wave exciting force in varying water depth

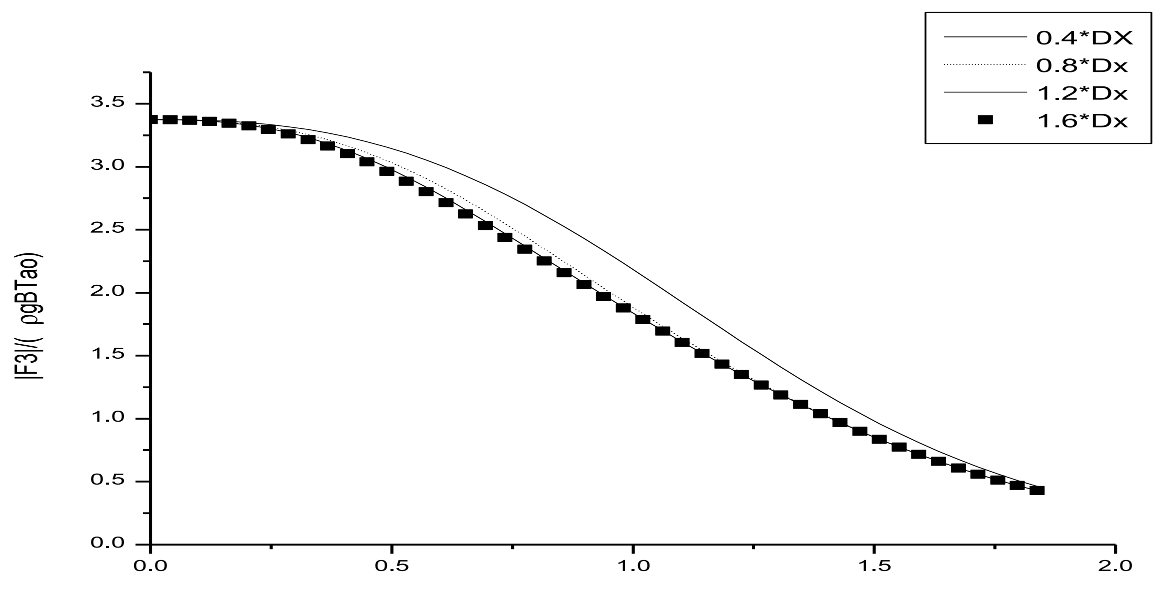

Figure 9: Graph showing the magnitude of heave wave exciting forces in varying water depth

In the present method, a rectangular floating barge was used in obtaining the hydrodynamic forces that is the Froude Krylov and wave exciting forces due to incidence waves. The dimensions of the box were taken as; Length, L = 2.25 m; Breadth, B = 2.25 m; and Draught, T = 1.00 m. The dimensions were taken as those of study done by Manyanga *et al.* (2014). This was to help in the comparison of two-layer and single layer cases. After writing a Fortran code containing all the information above and the mathematical formulation aforementioned, the graph in figure 6, figure 7, figure 8 and figure 9 were obtained. The graphs were in agreement with others that were obtained by other researchers (Endo, 1987; Manyanga *et al., 2014*). Figure 6 shows the magnitude of the surge wave exciting forces.

It is evident that the surge force increases significantly until it reaches the peak then it is radiated away at infinity. The forces decay to avoid interference. Figure 7 shows the magnitude of heave wave exciting which is very crucial to ocean engineers. The magnitude of the wave exciting force on the heave direction was inversely proportion to the wave frequency. At high frequency the forces and moments tend to approach zero that is, they decay. In addition when the dispersion relation was applied $\omega^2 = gk \tanh kh$ it showed that water depth had a significant effect on both heave and surge motion. It was evident that if the water depth was increased, the frequency also increased, as clearly shown in figure 9. This is in accordance with shallow water effect. It is clear from figure 9 that in shallow fluids the heave exciting forces is very high. This explains why waves generated by seismic disturbances are very catastrophic compared to other surface gravity waves, since tsunamis behave like shallow fluids. Consequently, the results predict that any offshore body would thrive well when the sea is deep.

From the results obtained it is possible to understand and appreciate the reason why harbors are constructed at very deep point near the shore. Moreover, offshore bodies do operate efficiently and safely in presence of low heave motion. Therefore from the results obtained there is need for very high wave frequency which lead to very low heave motion. Moreover, to reduce these forces the offshore body should be located at deeper from the free surface. As aforementioned figure 7 which shows the magnitude of the heave wave exciting force was in agreement with other results that had been used by other researchers Endo (1987) and Manyanga *et al.* (2014). However, comparing the surge and the heave wave exciting forces it was evident surge motions did not have a lot of impact on offshore bodies and this explain why it is not given a lot of attention by most researchers of hydrodynamic loads. However, Manyanga *et al.* (2014) analyzed the magnitude of the surge wave in two layer fluid and the two results were convergent.

From figure 8 it is evident that change in water depth has adverse effect on the magnitude of the wave exciting force. As the depth decreases the surge wave exciting force increases. The results that were obtained in this study were in agreement with others that had been established by Endo (1987) and Manyanga *et al.* (2014). Although the two study applied different methods to analyze waves characteristic the results were tremendous and convergent.

It was established that the use of Green Method developed by Wehausen and Laitone (1960) in its series form together with the Hess and Smith (1964) panel method were not only computationally simple but also very accurate in the analysis of the hydrodynamic loads. This is because the results obtained in this research were convergent to those of Endo (1987) and Manyanga *et al.* (2014)

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary

This study showed that it is possible and indeed important to analyze wave characteristics and consequently analyze their effects on a rectangular floating barge with high degree of accuracy, efficiency and confidence. It was evident that the study of surface waves and hydrodynamics loads is very critical. Therefore, there is need for them to be considered in the marine engineering designs and constructions. Furthermore, the system of equations that resulted from using the boundary integral method was in excellent agreement with results obtained by other researchers in similar studies.

5.2 Conclusions

This study achieved the objectives of solving the forces created by presence of an offshore body in wave terrain. By virtue of potential flow the study assumed that the fluid is incompressible and inviscid, and the flow is irrotational. In addition, by assuming that the problem is linear the study ignored the high order terms in all boundary conditions. However, these assumptions were valid and produced accurate results since the study only considered small perturbations about the body's mean position.

The complexity of the problem dictated that it is not possible to solve the velocity potential analytically and therefore there was need to develop a robust numerical scheme for the purpose of calculating the potential and forces acting on offshore bodies. The panel method was used in this respect where the body was divided into 192 panels. Many but small sized panels were used so as to reduce the deflection of wave due to the edges of the barge. Results showed that incident waves had adverse effects on the sea environment and on any floating structure on the wave field. For instance, it was observed that the vertical velocity and vertical acceleration significantly contributed to vertically induced motions. In this sense, there is need for ocean engineers to be knowledgeable on the wave characteristics. This would enable them design new and even modify the existing offshore structures in such a way that can withstand extreme wave fluctuations.

In this study Hess and Smith (1964) panel method was employed to investigate the hydrodynamic forces acting on a floating rectangular barge. The green functions that was applied in this study was in series form, hence it removed the points of singularity on the free surface unlike the integral green function used by Endo (1987). The results obtained could be used by engineers and designers of offshore structure in the field of oil exploitation to help them design structure with low heave motion. This would have the potential of increasing the amount of oil drilled at a time. From the research findings, this can be done by increasing the draught distance. The results obtained in this study were convergent to those of Endo (1987) and Manyanga *et al.* (2014).

5.3 Recommendations

This research analyzed the surface wave characteristics and wave exciting force acting on a rectangular floating barge at zero forward. The research could be extended in the following directions:

i Analysis of the hydrodynamic forces acting on a body moving in water at a given speed.

ii Investigations of radiation problems; that is, added mass, damping and restoring force acting on a freely moving body.

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